Thermodynamics of moist convection
Part I: heat engine and buoyancy

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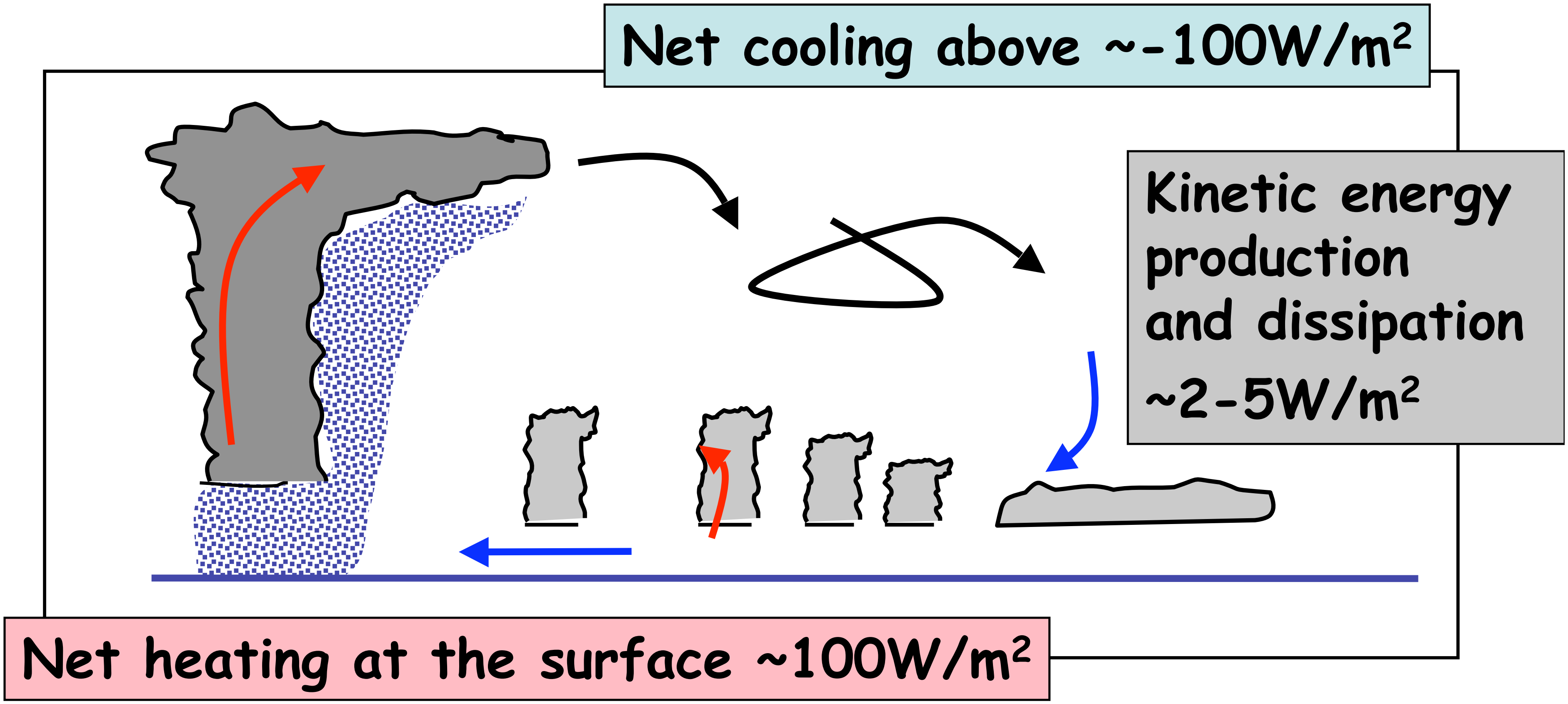
Outline

- Introduction: the nature of the problem…
- Entropy of moist air
- Heat engines and water vapor.
- Equation of state of moist air
- Idealized moist Rayleigh-Benard convection
- Buoyancy and entropy fluxes
- Isentropic circulation



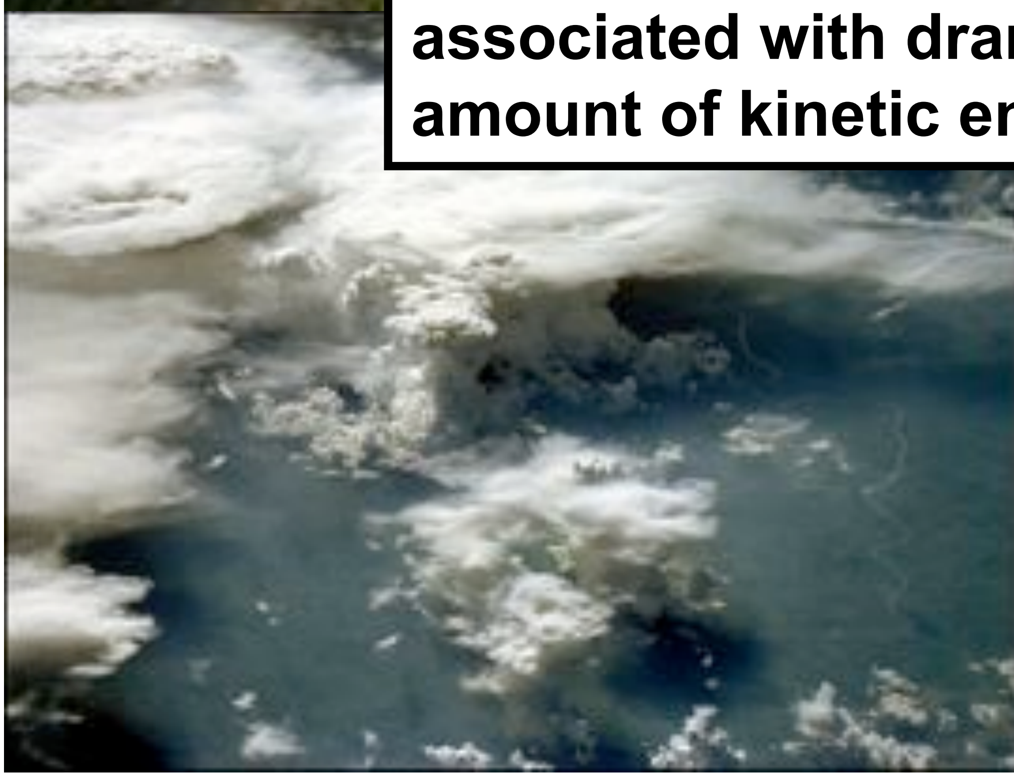
$$D \approx 4Wm^{-2}$$

$$\Rightarrow \varepsilon \approx 0.0004$$

$$\Rightarrow V_{1m} \sim 0.1ms^{-1}$$

But kinetic energy production and dissipation are highly intermittent!

Different weather regime are associated with dramatically different amount of kinetic energy production.



1A. Thermodynamics

- 1st Law - energy conservation

$$\Delta U + W = Q$$

- 2nd Law… what does it mean?

$$\Delta S = \frac{Q}{T} + \Delta S_{irr}$$

$$\Delta S_{irr} \geq 0$$

$$\Delta S = \frac{Q}{T} + \Delta S_{irr}$$

$$\Delta S_{irr} \geq 0$$

- S: entropy is a state variable.
- There are reversible processes.
- Entropy is conserved for reversible adiabatic transformations, i.e. two systems have the same entropy if they can be join by a sequence of reversible adiabatic transformation.
- But real transformations are irreversible and associated with a positive entropy production.

1B. Entropy of moist air

- Moist air can be treated as an ideal mixture of dry air, water vapor and liquid water. The entropy per unit mass of dry air S is then:

$$S = s_d + r s_v + r_l s_l$$

With S: entropy per unit mass of dry air;

r: mixing ratio (water vapor concentration)

r_l: mixing ratio for condensed water

$r_T = r + r_l$: mixing ratio for total water

$s_d, s_v . s_l$: specific entropy for dry air, water vapor and liquid water

- The specific entropies are defined up to an additive constant:

$$s_d = C_{pd} \ln \frac{T}{T_o} - R_d \ln \frac{p_d}{p_o} + s_{d0}$$

$$s_v = C_{pv} \ln \frac{T}{T_o} - R_v \ln \frac{e}{e_o} + s_{v0}$$

$$s_l = C_l \ln \frac{T}{T_o} + s_{l0}$$

- We cannot put all the integration constant to 0 because the entropy of water vapor and liquid water must be such that:

$$s_v - s_l = \frac{L_v}{T} \text{ at saturation } (e = e_s(T) \text{ or } H = 1)$$

- 'Moist entropy *S*': set $s_{l0} = s_{d0} = 0$

$$\boxed{\rightarrow S = (C_{pd} + r_T C_l) \ln \frac{T}{T_0} + R_d \ln \frac{p_d}{p_0} + r\left(\frac{L_v}{T} - R_v \ln H\right)}$$

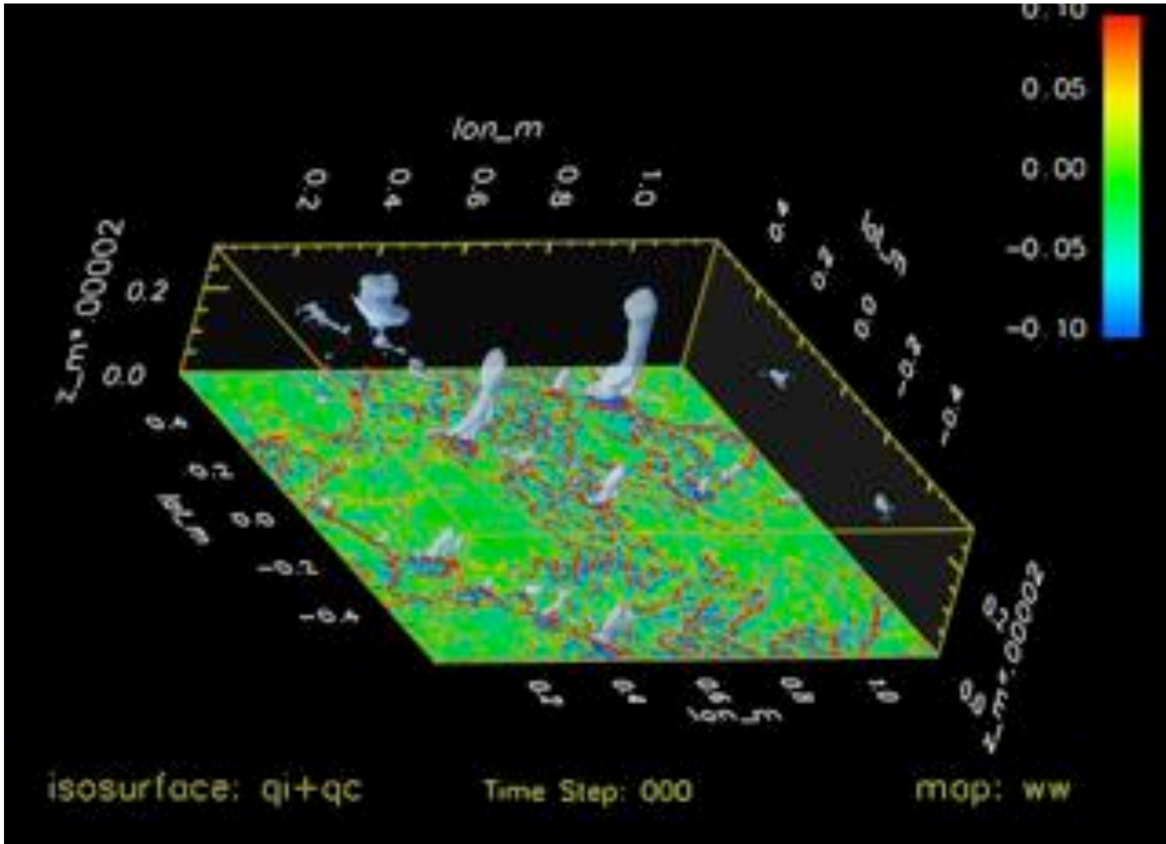
A brief note on adiabatic invariants:

- The thermodynamic properties of dry air can be described by 2 state variables, say entropy and pressure. As pressure is not invariant, any adiabatic invariant is function of entropy alone.

$$\boxed{\frac{dF(S)}{dt} = 0 \text{ for reversible adiabatic transformations.}}$$

- For moist air, we need at least three state variables, e.g. entropy, total water concentration, and pressure. Any function of entropy and total water content is an adiabatic invariant:

$$\boxed{\frac{dF(S,r_T)}{dt} = 0 \text{ for reversible adiabatic transformations.}}$$


lon_m
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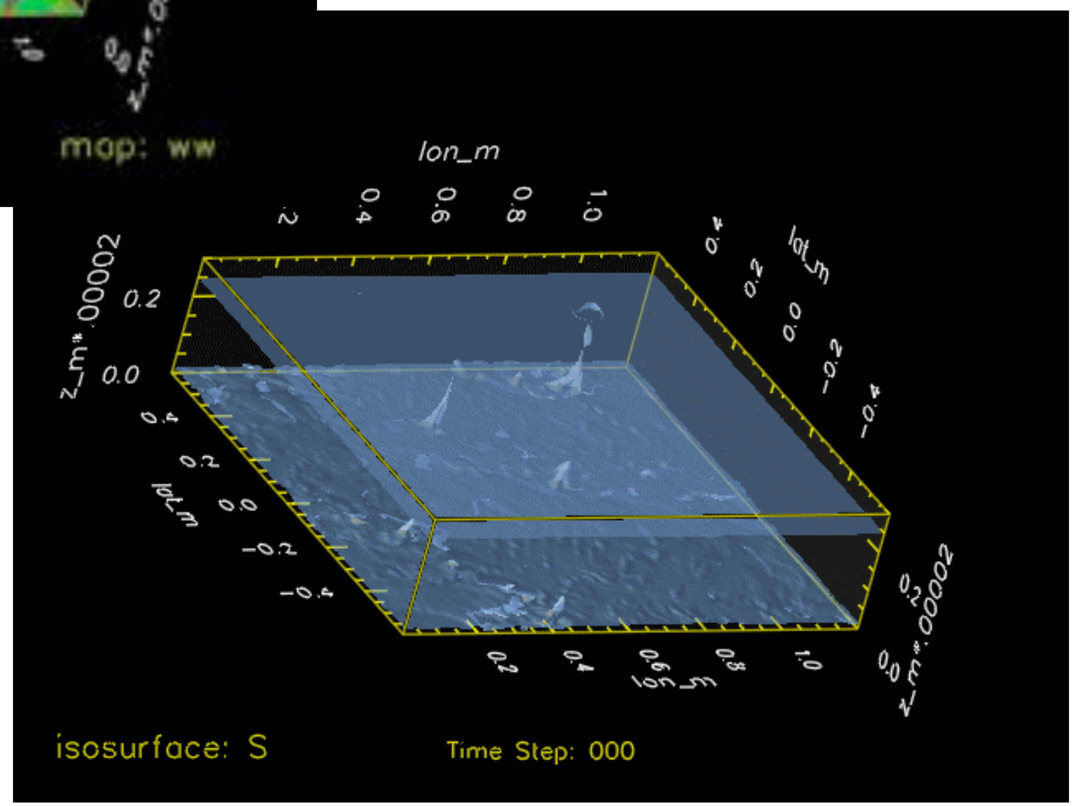
0.05
0.00
-0.05
-0.10
isosurface: qi+qc
Time Step: 000
map: ww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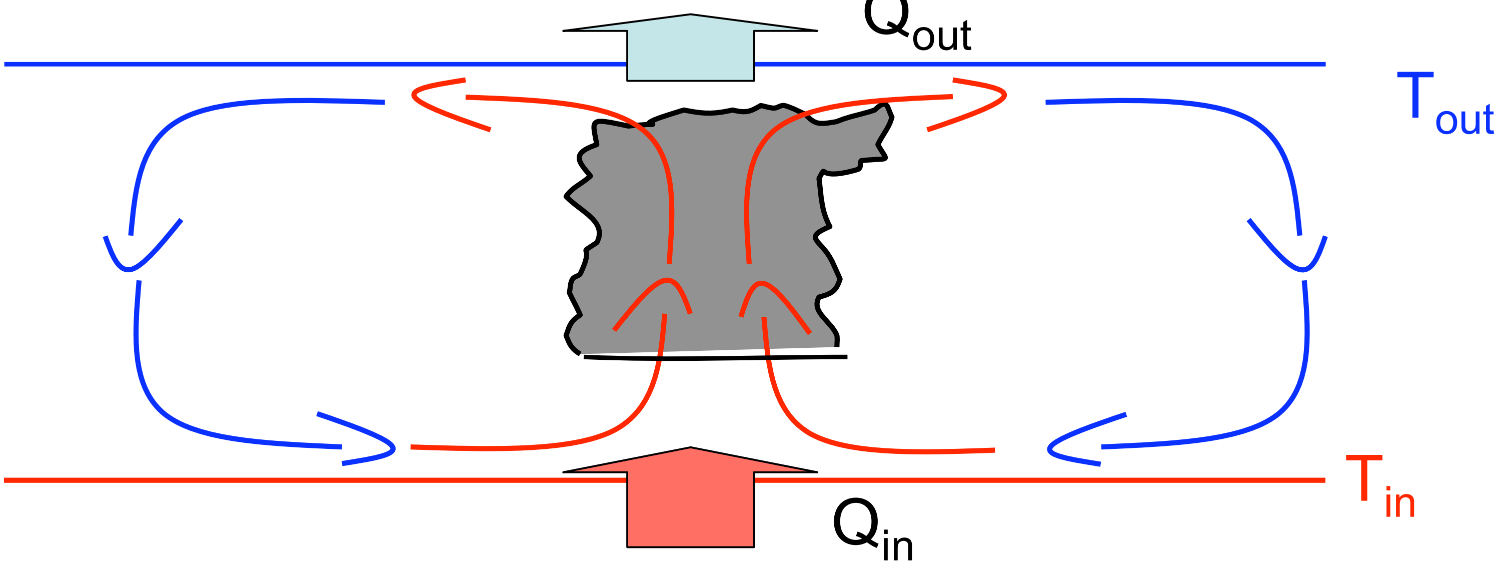
Time Step: 000

2. Idealized heat engine



- Idealized problem: convection transport water vapor and energy upward from a warm/moist source to a dry/cold sink.
- Situation is analogous to shallow, non-precipitating convection.

2a. Carnot cycle

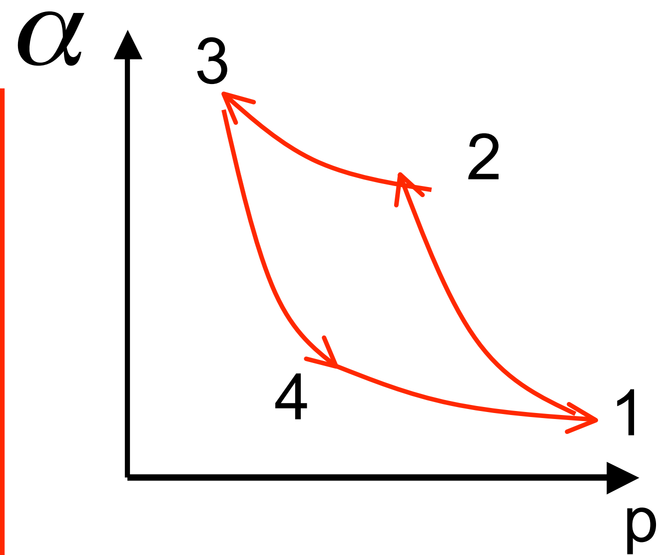
$1 \rightarrow 2:$ isothermal expansion at T_{in}

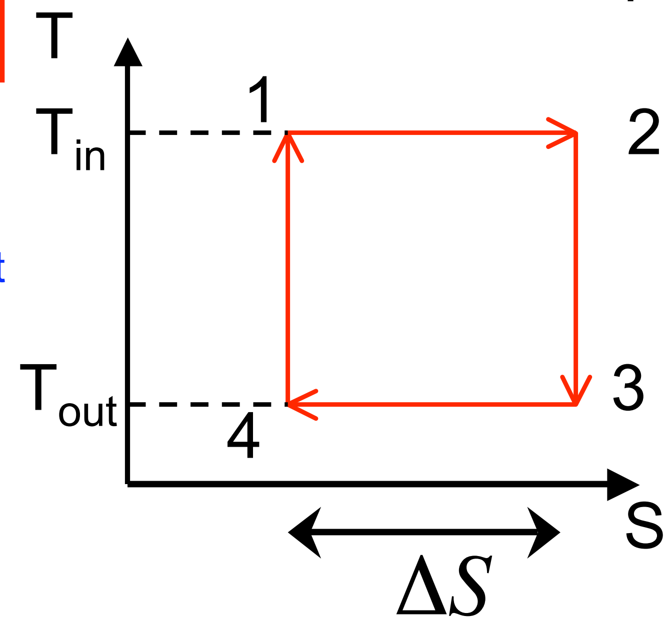
$2 \rightarrow 3:$ adiabtic expansion with $S_2 = S_3$

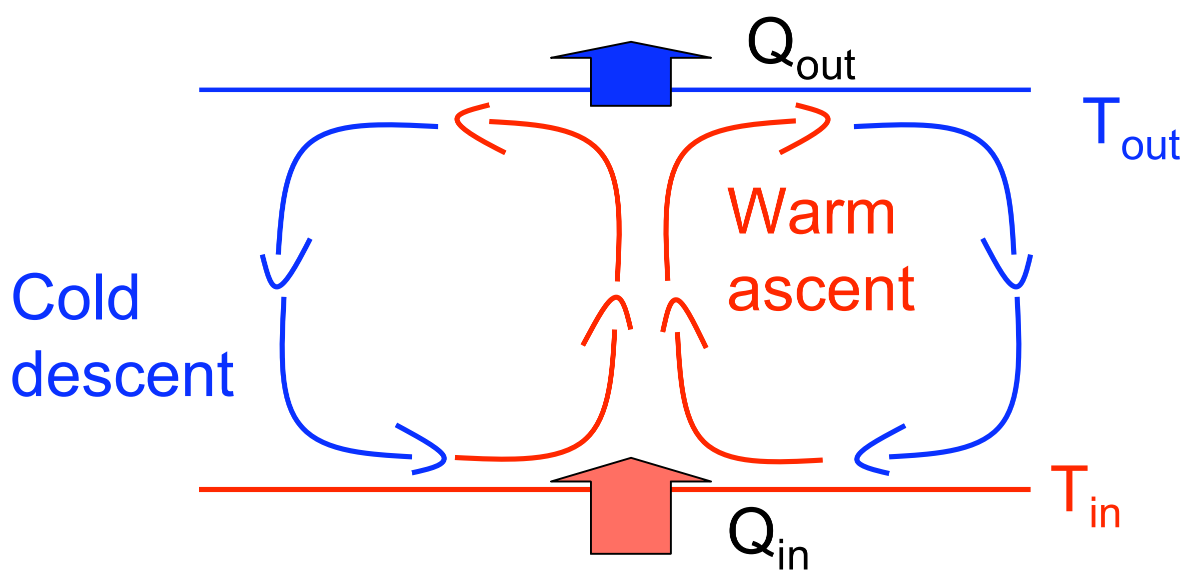
$3 \rightarrow 4:$ isothermal compression at T_{out}

$4 \rightarrow 1:$ adiabatic compression with $S_2 = S_3$

Total water content is constant through the entire cycle!







- Mechanical work is defined as

$$W = \oint -\alpha(S, r_T, p)\,dp$$

- Using the thermodynamic relationship

$$TdS = dh - \alpha dp$$

we get:

$$W = \oint TdS = (T_{in} - T_{out})\Delta S$$

- External heating

$$\delta Q = dh - \alpha dp = TdS$$

- Heating at the warm source:

$$Q_{in} = \oint \delta Q^{+} = \int_{1}^{2} TdS = T_{in}\Delta S$$

$$\boxed{\text{Efficieny } \eta_C = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}}}$$

2B. Steam cycle

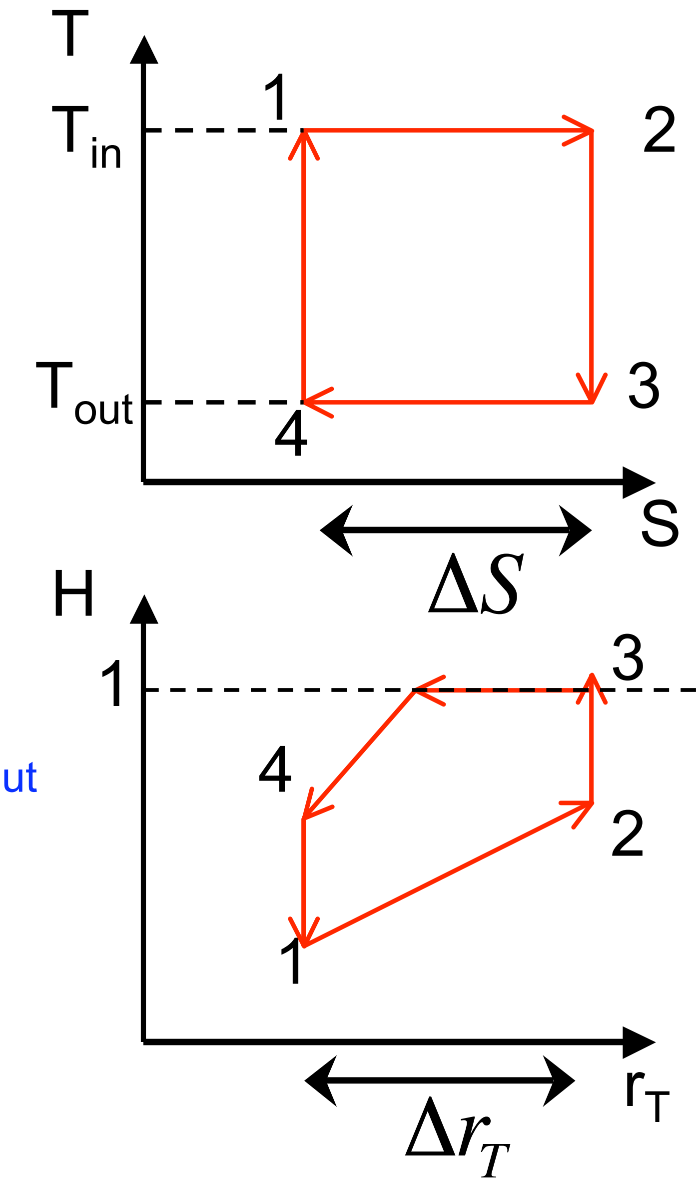
$1 \rightarrow 2:$ isothermal moistening at T_{in}

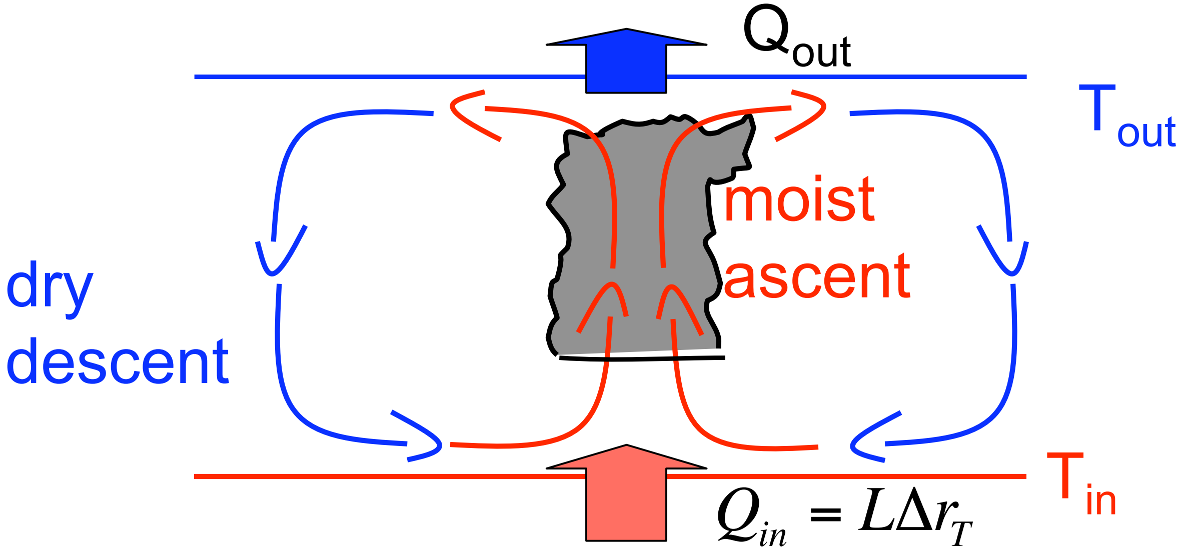
$2 \rightarrow 3:$ adiabtic expansion

$3 \rightarrow 4:$ isothermal drying at T_{out}

$4 \rightarrow 1:$ adiabatic compression

Heating is due solely to evaporation!





The expression

$$TdS = dh - \alpha dp$$

is only valid for close transformations.

To account for the addition or removal of mass, we need an additional term:

$$TdS = dh - \alpha dp - g dr_T$$

where g is the Gibbs free energy of water vapor per unit of mass:

$$\begin{aligned} g &= h - Ts \\ &= C_{pv}(T - T_0 - \ln\frac{T}{T_0}) + R_v T \ln H \\ &\approx R_v T \ln H \end{aligned}$$

- Mechanical work:

$$\boxed{\begin{aligned} W &= \oint T dS + \oint g dr_T \\ &= (T_{in} - T_{out})\Delta S + (g_{in} - g_{out})\Delta r_T \end{aligned}}$$

- with

$$g_{in} = R_v T_{in} \ln H_{in}$$

$$g_{out} = R_v T_{out} \ln H_{out}$$

- Surface heating: $Q_{in} = T_{in}\Delta S + g_{in}\Delta r_T = L\Delta r_T$

- Entropy change: $\Delta S = \left(\frac{L - g_{in}}{T_{in}}\right)\Delta r_T$

$$\boxed{\text{Efficieny } \eta_H = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}} + \frac{R_v T_{out}}{L} \ln \frac{H_{in}}{H_{out}}}$$

$$\text{Efficieny } \eta_H = \frac{T_{in} - T_{out}}{T_{in}} + \frac{R_v T_{out}}{L} \ln \frac{H_{in}}{H_{out}}$$

Carnot efficiency

additional term depending on saturation

- The efficiency depends on the state of the system!!!
- Saturated case: *H=1*

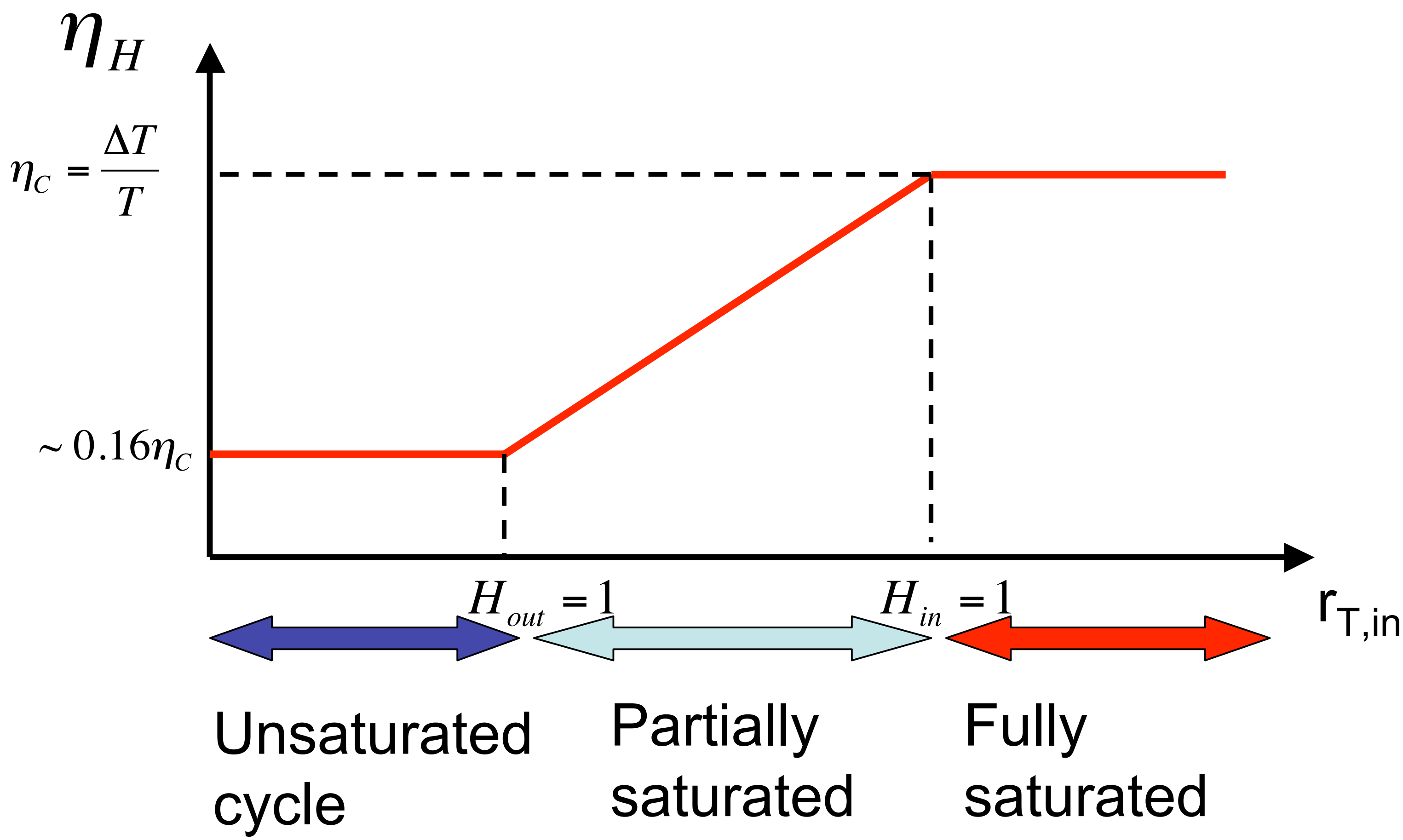
$$\text{Efficieny } \eta_{H,sat} = \frac{W}{Q_{in}} = \frac{T_{in} - T_{out}}{T_{in}}$$

- General case: the relative humidity increases with height, i.e

$$\boxed{H_{out} \geq H_{in} \rightarrow \eta_H \leq \frac{T_{in} - T_{out}}{T_{in}}}$$

- Hence, a steam cycle produces at best as much mechanical work as a Carnot cycle

- Three regimes:
  - The cycle is unsaturated at all time: efficiency is minimum.
  - The cycle is partially saturated: efficiency increases with amount of water in the cycle.
  - The cycle is saturated at all time: efficiency is maximum and given by the Carnot efficiency



2C. There is no free lunch…

- Its rate of change is given by

$$dg = sdT + \alpha dp$$

- For a reversible isothermal process, we have:

$$\boxed{\begin{aligned} dg - \alpha dp &= 0 \\ \Rightarrow \Delta g + W &= 0 \end{aligned}}$$

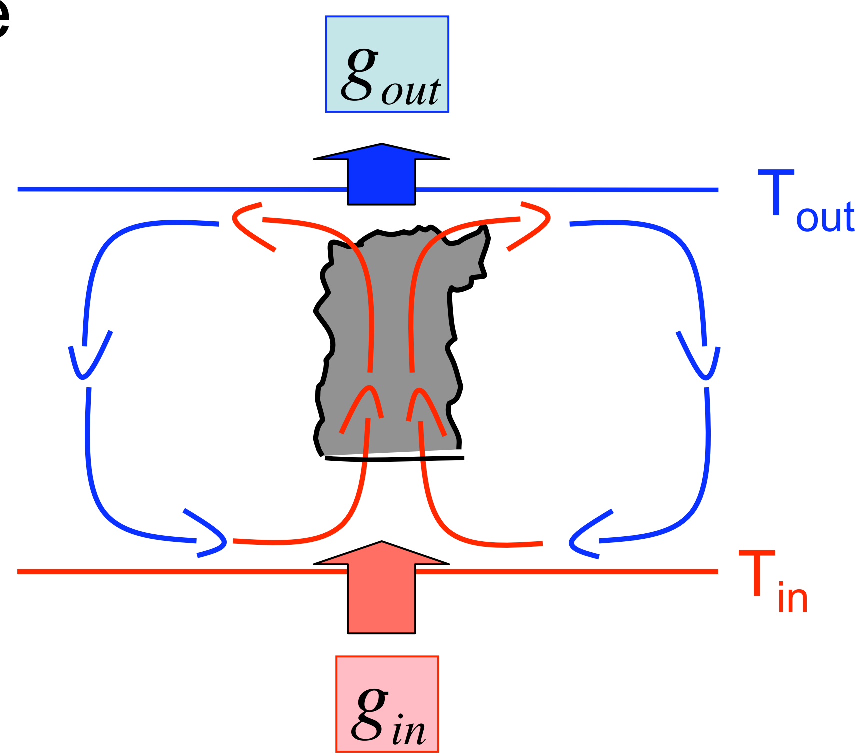
- The amount of work that can be extracted is equal to the reduction in free energy!
- And it is only possible to increase the free energy if work is exerted on the system

"water vapor transport penalty" due to an increase in the free energy as water is transported upward

$$\left(g_{v,in} - g_{v,out}\right)\rho_0 \overline{w' r_T{}'}$$

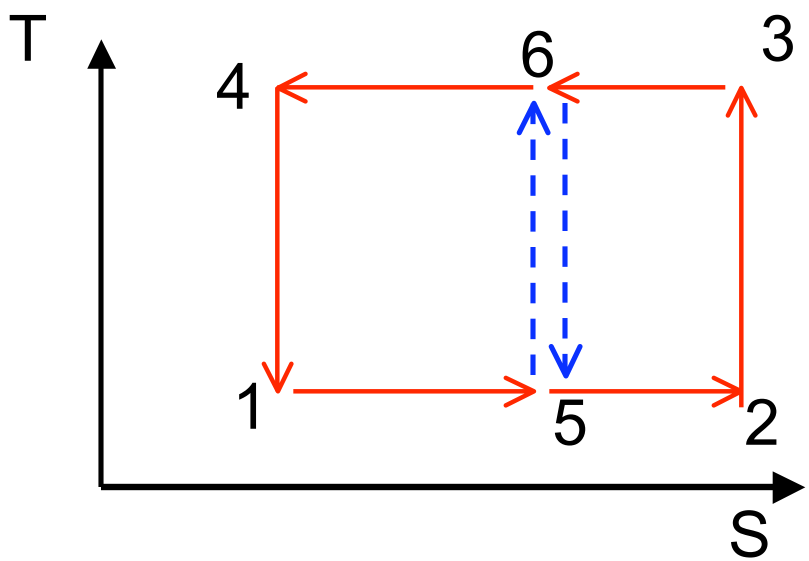
$$g_v = R_v T \ln H$$

- Free energy increase with height in an unsaturated ascent
- Bur is constant whenever the air is saturated



2d. Mixed Carnot-steam cycle

$1 \rightarrow 2:$ isothermal heating
and moistening at T_{in}
$2 \rightarrow 3:$ adiabtic expansion
$3 \rightarrow 4:$ isothermal cooling
and drying at T_{out}
$4 \rightarrow 1:$ adiabatic compression



- Intermediary steps 5 and 6 such that cycle 1-5-6-4 is a humidifier and 5-2-3-6 is a Carnot cycle.

- Latent and sensible heat flux:

$$Q_{lat} = L\Delta r_T$$

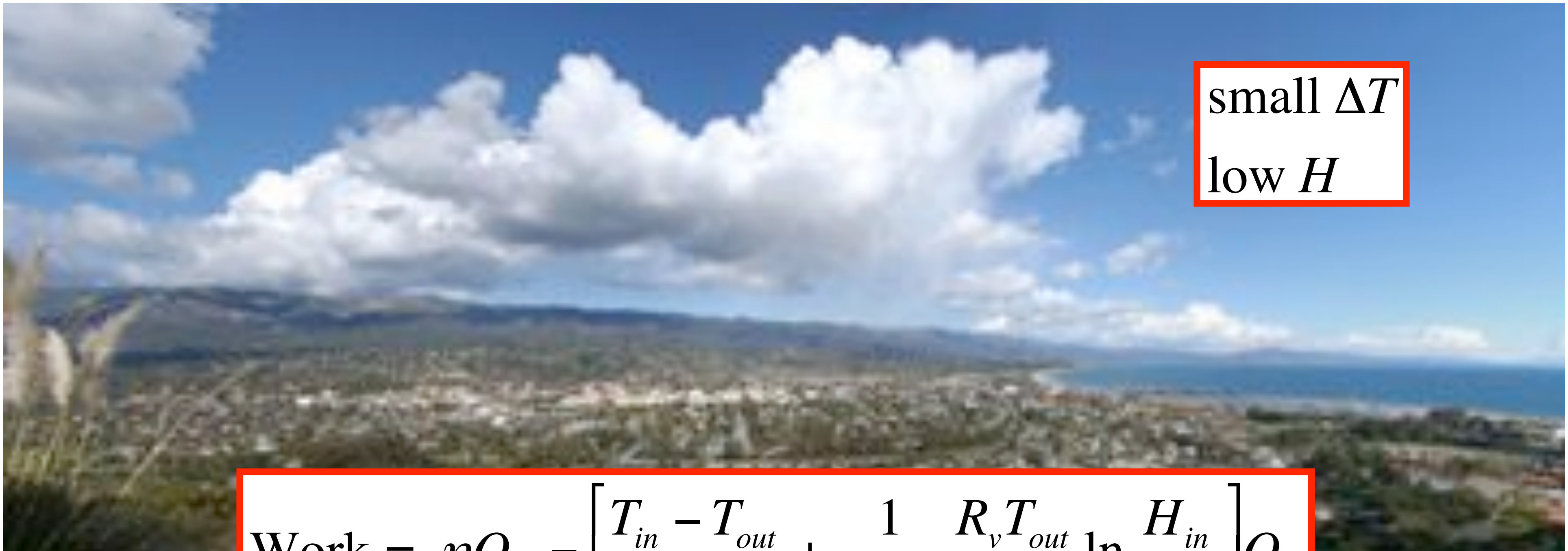
$$Q_{sen} = T_{in}\Delta S + (g_{in} - L)\Delta r_T$$

- Bowen ratio:

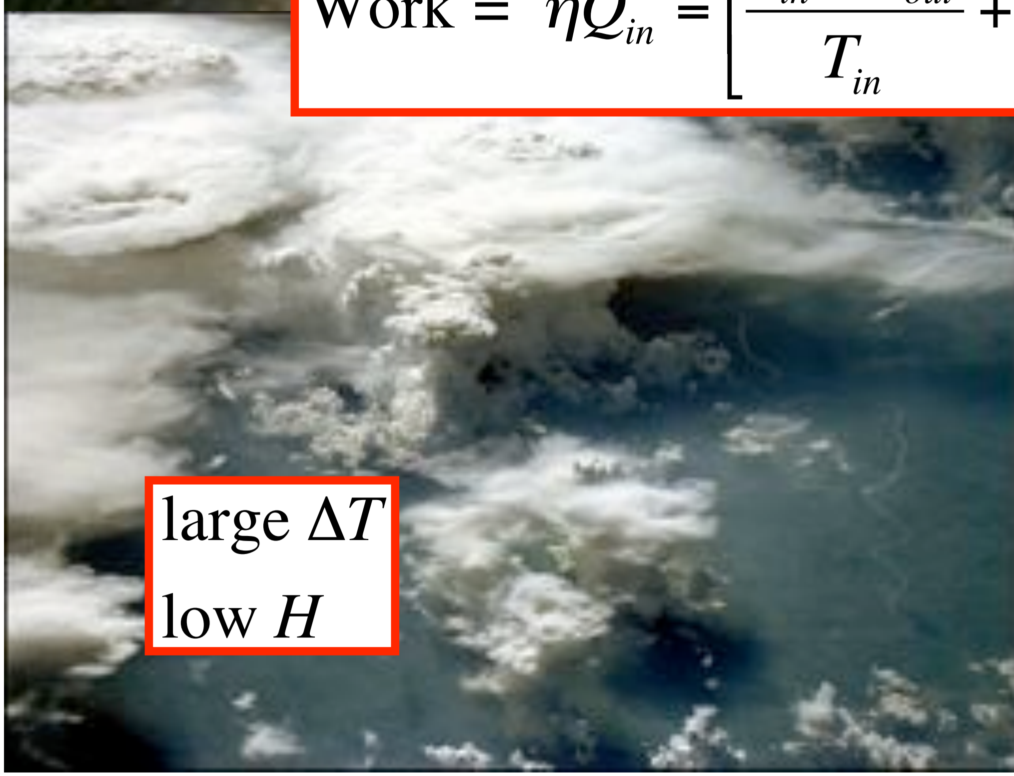
$$B = \frac{Q_{sen}}{Q_{lat}}$$

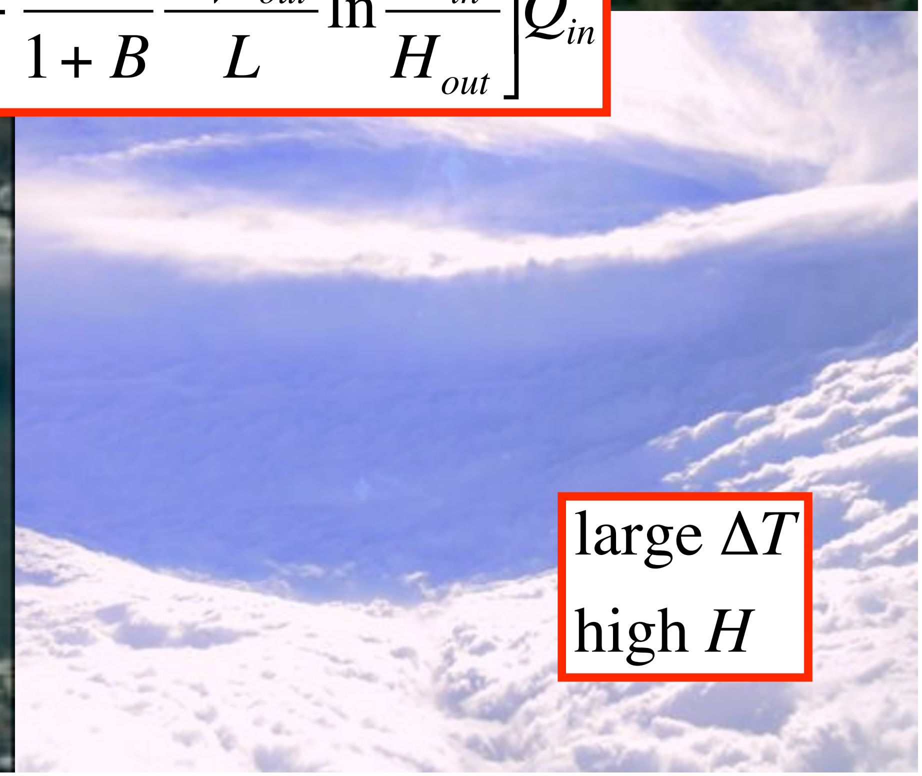
$$\boxed{\begin{aligned}\text{Efficieny } \eta &= \frac{B}{1+B}\eta_c + \frac{1}{1+B}\eta_H \\ &= \frac{T_{in} - T_{out}}{T_{in}} + \frac{1}{1+B}\frac{R_v T_{out}}{L}\ln\frac{H_{in}}{H_{out}}\end{aligned}}$$

The efficiency of an atmospheric heat engine depends on both its degree of saturation and on the Bowen ratio.



$$\text{Work} = \eta Q_{in} = \left[\frac{T_{in} - T_{out}}{T_{in}} + \frac{1}{1+B}\frac{R_v T_{out}}{L}\ln\frac{H_{in}}{H_{out}}\right]Q_{in}$$





3A. Equation of state for moist air

- Equation of state relates various thermodynamic properties (I.e state variable) of a fluid.
- We are particularly interested in expressed properties like specific volume in terms of adiabatic invariants and pressure.
- Moist air is treated as a mixture of dry air, water vapor and condensed water. Water vapor and dry air are treated as ideal gases. Liquid water is treated as incompressible and its volume is neglected.

Specific volume:

- We start from the ideal gas law

$$PV = (N_d + N_v)R_* T$$
$$= \left(N_d m_d\right)\frac{R_*}{m_d}T + \left(N_v m_v\right)\frac{R_*}{m_v}T$$

- Then, after some reorganization, we get:

$$\alpha = \left[\frac{1}{1+r_T}R_d T + \frac{r}{1+r_T}R_v\right]\frac{T}{P}$$
$$= \left(\frac{1+{}^{R_v}\!/\!_{R_d}\, r}{1+r_T}\right)\frac{R_d T}{P} = \frac{R_d T_\rho}{P}$$

- At the end, we can express the specific volume as a (smooth) function of four state variables, i.e.:

$$\alpha = \alpha(p, R, r, r_T)$$

Thermodynamic equilibrium

- Condensed water can only be present if it is in thermodynamics equilibrium with water. I.e we have

$$\boxed{\begin{array}{ll} \text{either } e \le e_s(T) \text{ and } r_l = 0 & \text{(unsaturated air)} \\ \text{or } e = e_s(T) \text{ and } r_l \ge 0 & \text{(saturated air)} \end{array}}$$

e : partial pressure of water vapor

e_s : saturation vapor pressure

T : Temperature

q_l : concentration of condesate water

Phase transition and partial derivatives

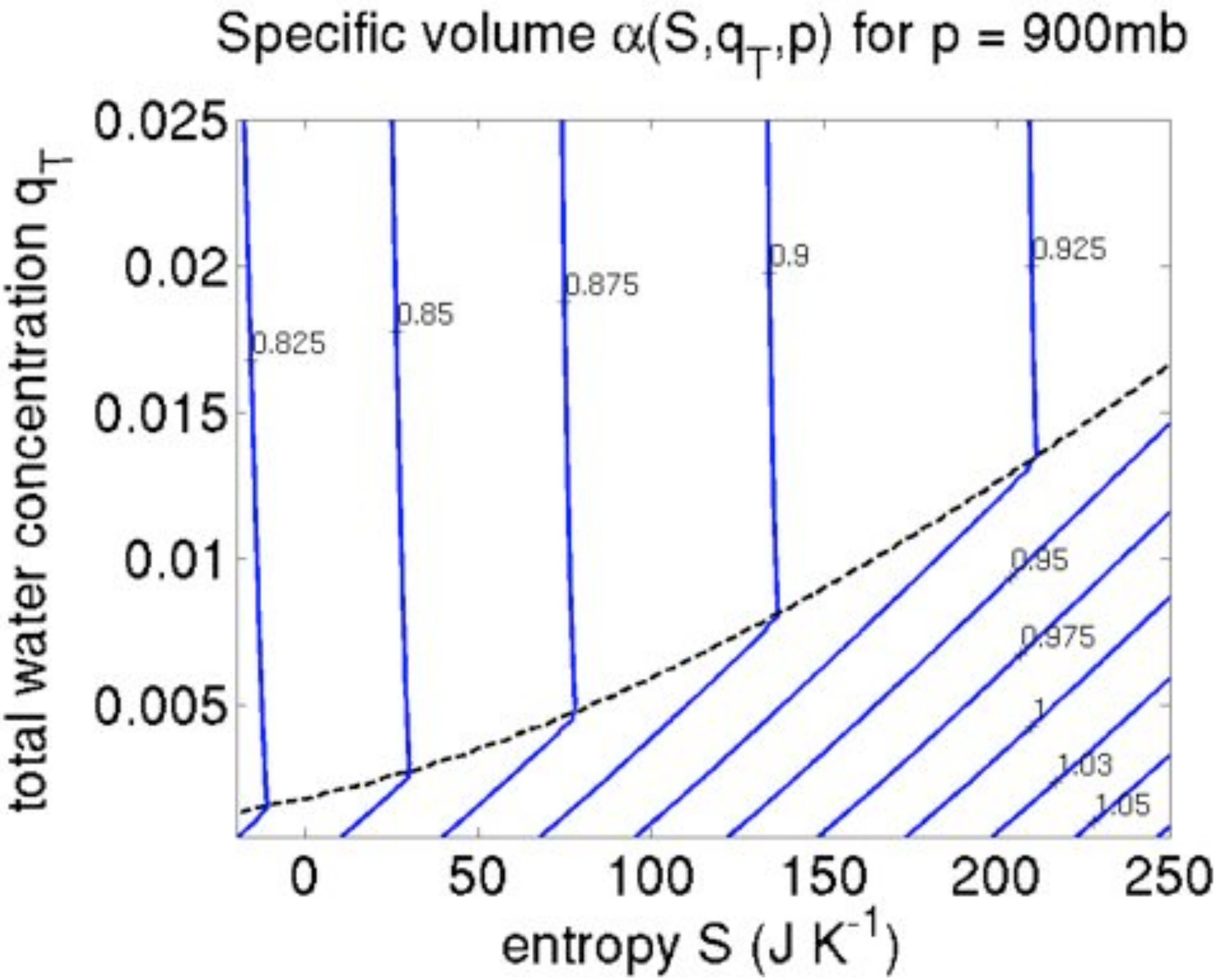
- Thermodynamic equilibrium introduces a switch condition in the description of the state of moist air:

$$r_l = \begin{cases} 0 & \text{for } r_T < r_s(T,p) \\ r_T - r_s(T,p) & \text{for } r_T < r_s(T,p) \end{cases}$$

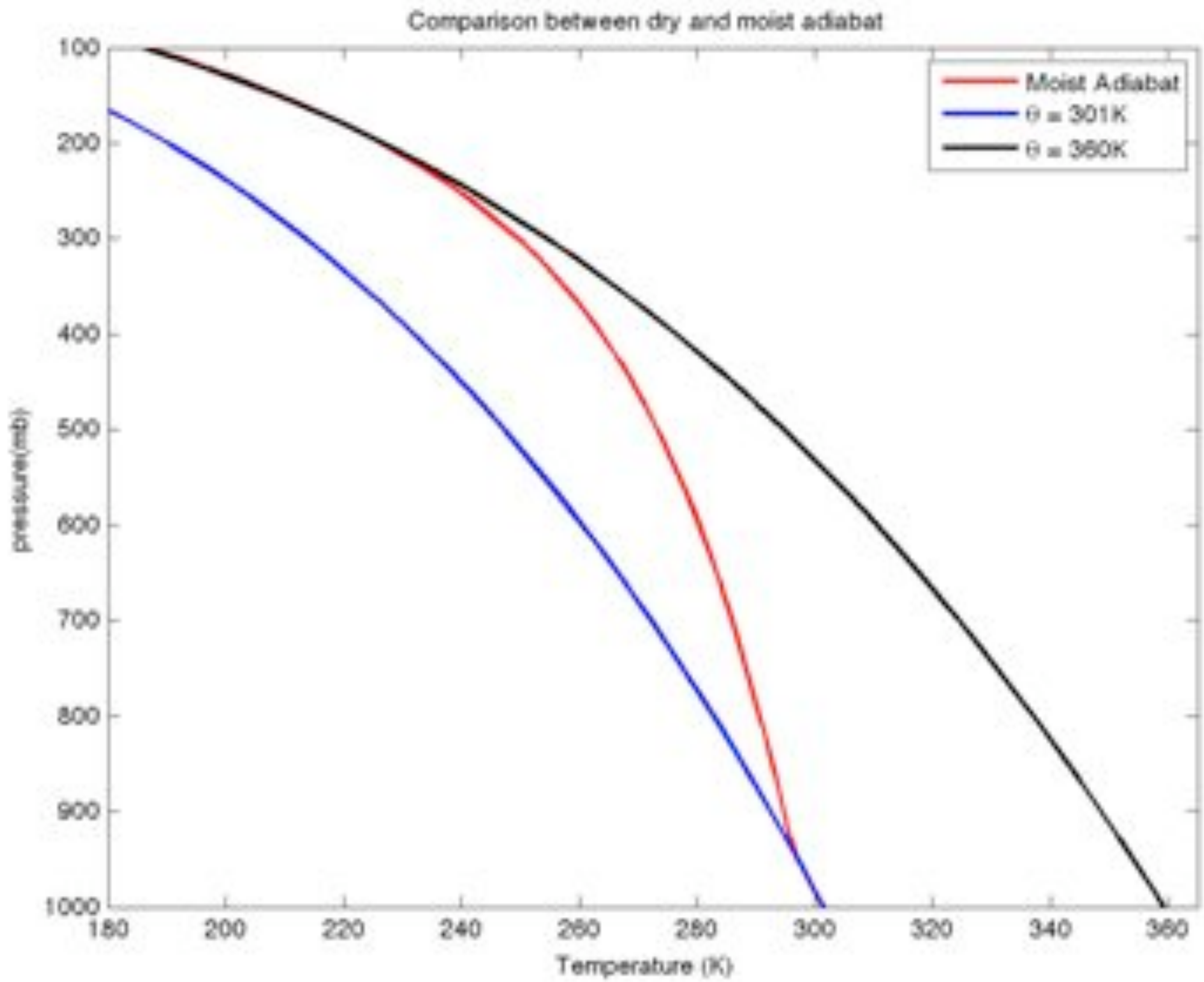
- This implies a discontinuity in the partial derivatives of the equation of state:

$$\left(\frac{\partial r_l}{\partial r_T}\right)_{p,T} = \begin{cases} 0 & \text{for } r_T \leq r_s(T,p) \\ 1 & \text{for } r_T > r_s(T,p) \end{cases}$$

- This applies not only to liquid water content, but to almost all states variables.



Specific volume α(S,q_T,p) for p = 900mb
total water concentration q_T
entropy S (J K^-1)
0.025
0.02
0.015
0.01
0.005
0
50
100
150
200
250
0.825
0.85
0.875
0.9
0.925
0.95
0.975
1
1.03
1.05



$$\text{Adiabatic Lapse Rate} : \Gamma = -\rho g \left(\frac{\partial T}{\partial p} \right)_{S, r_T}$$

3B. Boussinesq Approximation

- Start with the compressible N-S equation

$$\frac{du}{dt} = -\alpha \nabla p - g\mathbf{k} - \nu \nabla^2 u$$

$$\frac{d\alpha}{dt} - \alpha \nabla \bullet u = 0$$

- Assume an isentropic reference state
- Expand for small perturbation in pressure and density:

$$\frac{du}{dt} = -\nabla P + B\mathbf{k} - \nu \nabla^2 u$$

$$\nabla \bullet u = 0$$

Variations of density only enter through the buoyancy term in the vertical momentum equation:

$$B = B(X_1 \ldots X_n, z) = g\frac{\alpha(X_1 \ldots X_n, p_{ref}(z)) - \alpha_{ref}(z)}{\alpha_{ref}(z)}$$

where the specific volume α

$$\alpha = \alpha(X_1 \ldots X_n, p)$$

is a non-linear function of the pressure *p* and other state variables $X_1 \ldots X_n$.

The simplest choice is then to choose $X_1 \ldots X_n$ to be adiabatic invariants:

$$\frac{dX_i}{dt} = \kappa \nabla^2 X_i$$

What about the adiabatic invariants?

- We use the entropy S and the total water concentration q_T as adiabatic invariants:

$$\frac{dS}{dt} = \dot{S} + \kappa \nabla^2 S$$

$$\frac{dq_T}{dt} = \dot{q_T} + \kappa \nabla^2 q_T.$$

- The buoyancy is a non-linear function of the two invariants:

$$B(S, q_T, z) = g \frac{\alpha(S, q_T, p_{ref}(z)) - \alpha_{ref}(z)}{\alpha_{ref}(z)}.$$

Piece-wise linear equation of state:

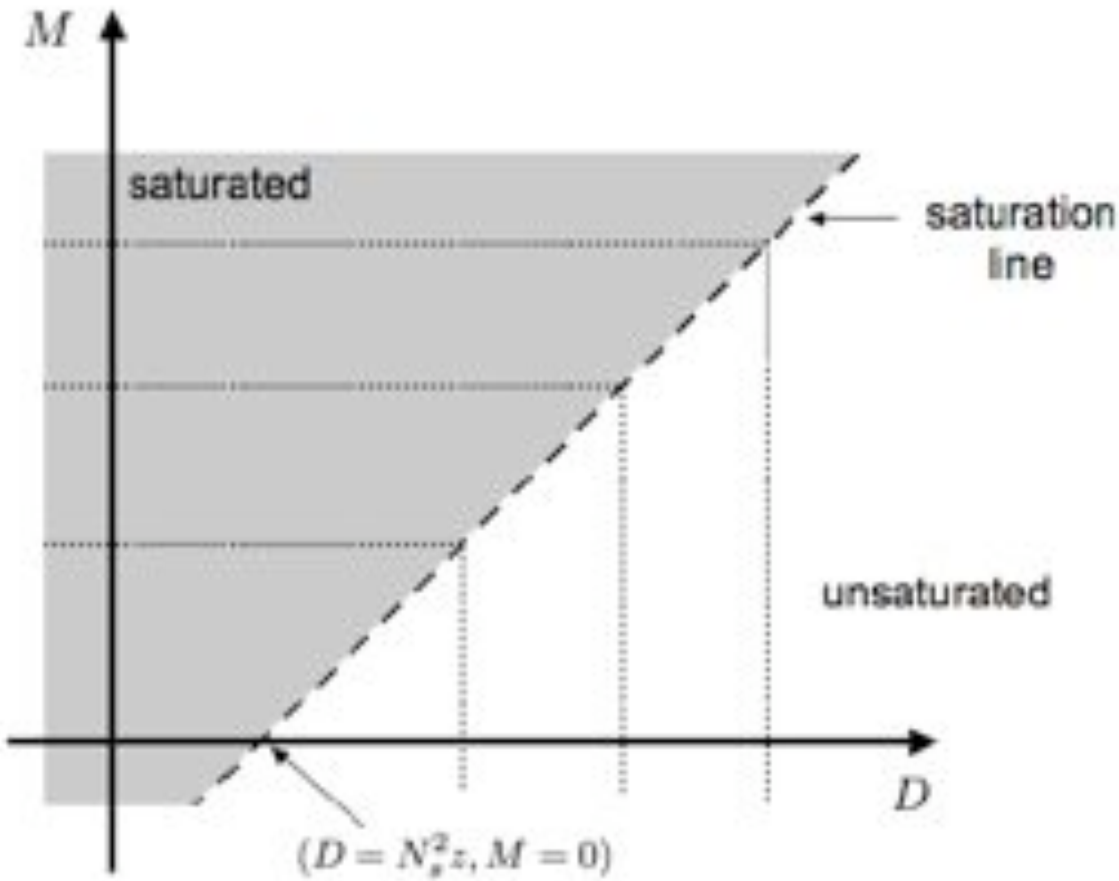
Linearize the equation of state within the saturated and unsaturated regions:

$$\left(\frac{\partial B}{\partial S}\right)_{q_T,z} = \frac{g}{\alpha_{ref}}\left(\frac{\partial \alpha}{\partial S}\right)_{q_T,p} = \begin{cases} B_{S,u} & \text{if } q_T \leq q_{sat}(S,z) \\ B_{S,s} & \text{if } q_T > q_{sat}(S,z) \end{cases}$$

$$\left(\frac{\partial B}{\partial q_T}\right)_{S,z} = \frac{g}{\alpha_{ref}}\left(\frac{\partial \alpha}{\partial q_T}\right)_{S,p} = \begin{cases} B_{q_T,u} & \text{if } q_T \leq q_{sat}(S,z) \\ B_{q_T,s} & \text{if } q_T > q_{sat}(S,z). \end{cases}$$

Introduce two new variables, the ‘dry’ and ‘moist’ buoyancies *D* and *M*:

$$D = B_{S,u}(S - S_{ref}) + B_{q_T,u}(q_T - q_{T,ref})$$
$$M = B_{S,s}(S - S_{ref}) + B_{q_T,s}(q_T - q_{T,ref}).$$



- For unsaturated parcels, $\left(\frac{\partial B}{\partial D}\right)_{M,z} = 1$ and $\left(\frac{\partial B}{\partial M}\right)_{D,z} = 0,$
- While for saturated parcels, $\left(\frac{\partial B}{\partial D}\right)_{M,z} = 0$ and $\left(\frac{\partial B}{\partial M}\right)_{D,z} = 1.$

- We still need a condition for saturation:

$$M - D \geq -N_s^2 z$$

- The full system is then

$$\frac{d\mathbf{u}}{dt} = -\nabla p' + B\mathbf{k} + \nu\nabla^2\mathbf{u}$$

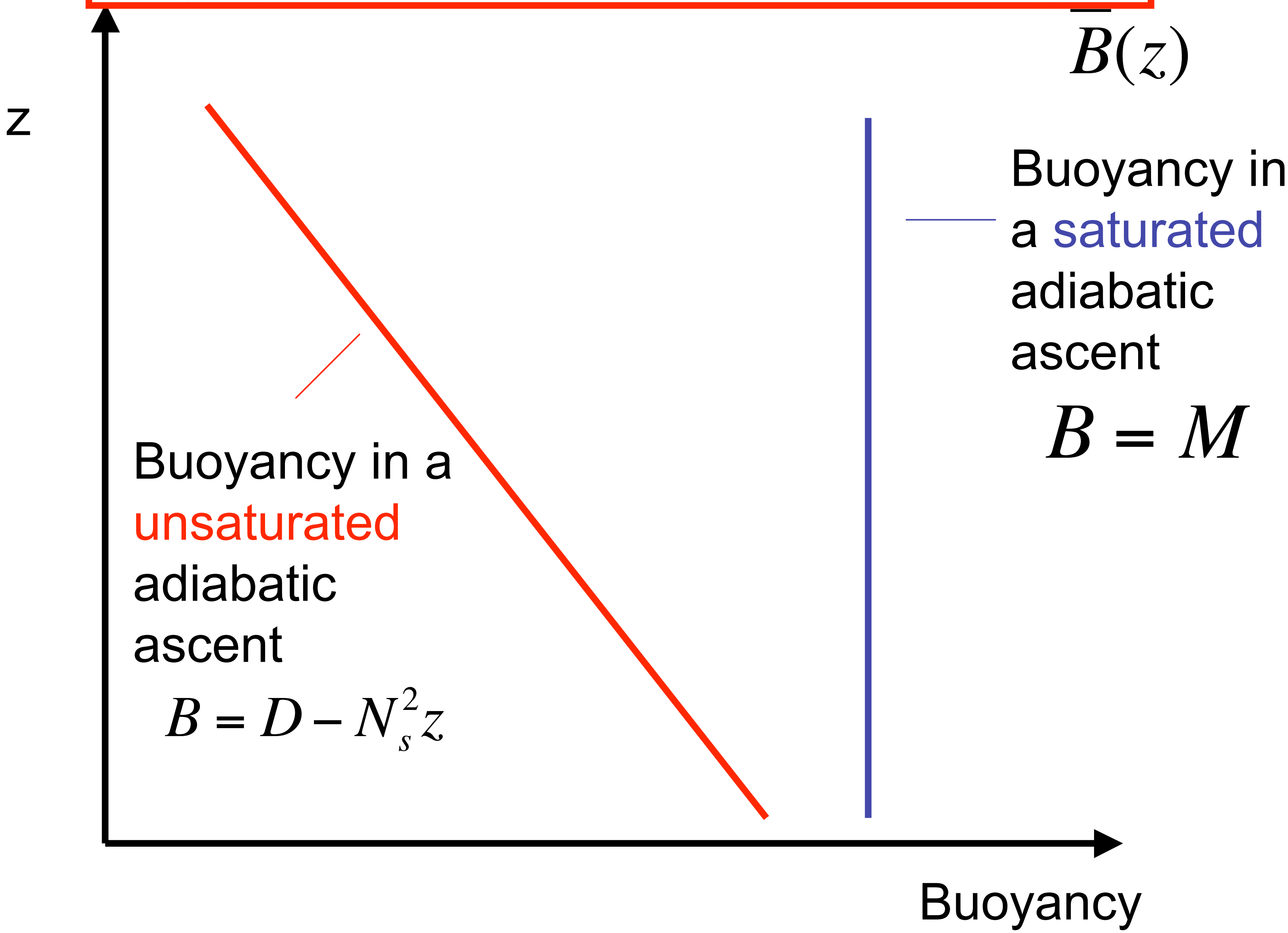
$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{dD}{dt} = \dot{D} + \kappa\nabla^2 D$$

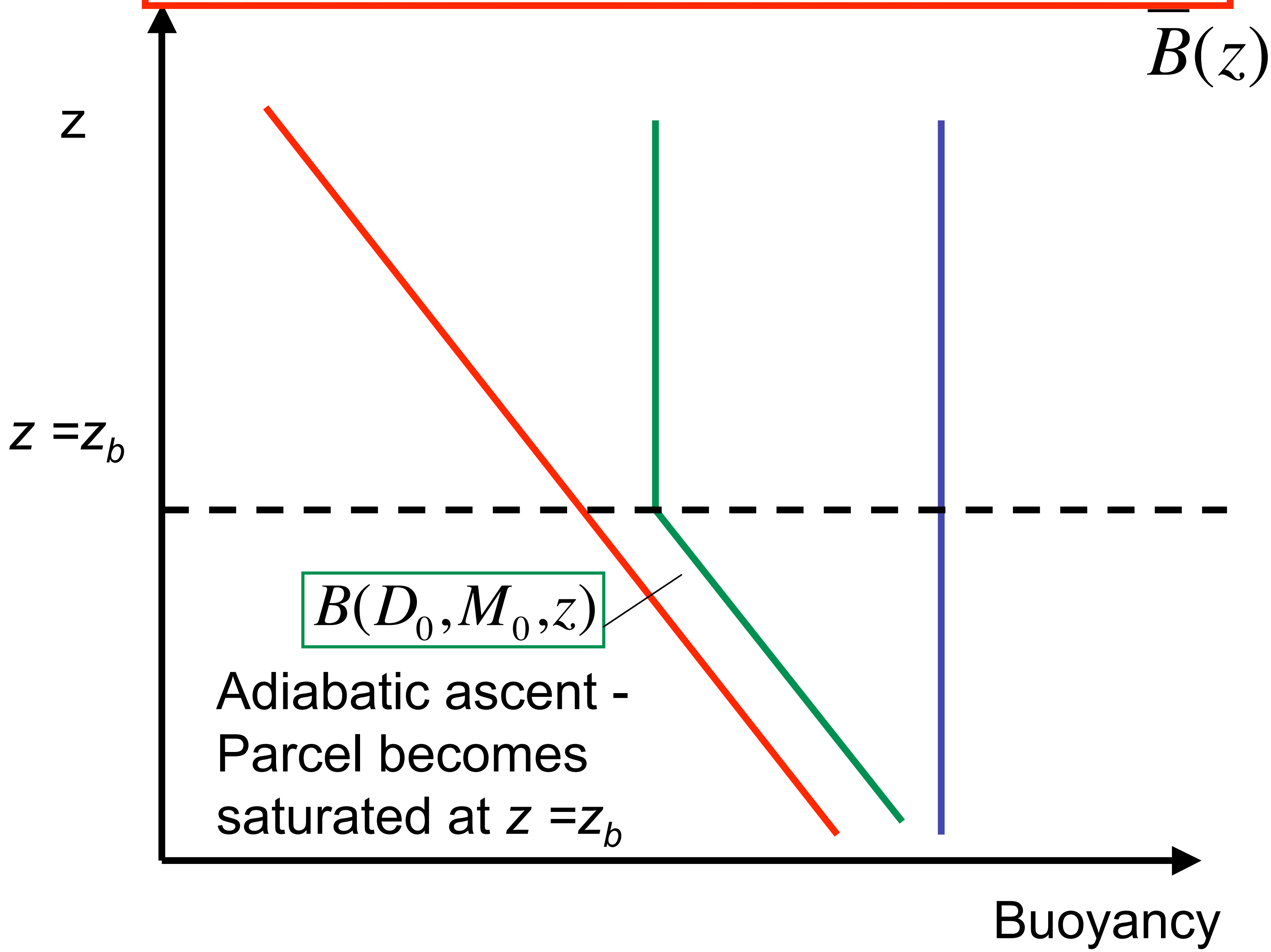
$$\frac{dM}{dt} = \dot{M} + \kappa\nabla^2 M$$

$$\boxed{B(D, M, z) = \max(M, D - N_s^2 z)}$$

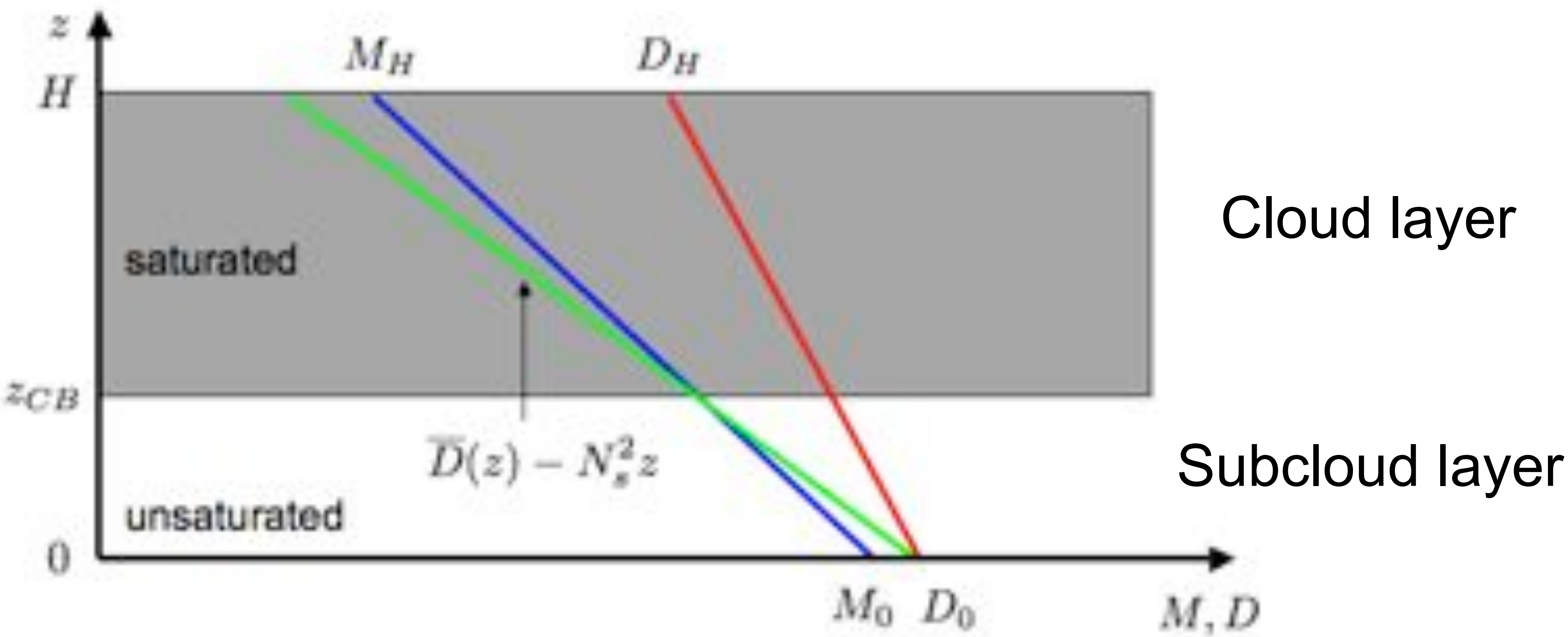
$$\boxed{B(D, M, z) = \max(M, D - N_s^2 z)}$$



$$\boxed{B(D, M, z) = \max(M, D - N_s^2 z)}$$



Moist Rayleigh-Benard convection



Analog to the classic Rayleigh-Benard convection but now, both the 'dry' and 'moist' buoyancyes *D* and *M* must be specified at the upper and lower boundary

5 non-dimensional parameters

$$\frac{d\mathbf{u}^*}{dt^*} = -\nabla_* p^* + B^*(M^*, D^*, z^*)\mathbf{k} + \sqrt{\frac{Pr}{Ra_M}}\nabla_*^2 \mathbf{u}^*$$

$$\nabla_* \cdot \mathbf{u}^* = 0$$

$$\frac{dD'^*}{dt^*} = \frac{1}{\sqrt{Pr Ra_M}}\nabla_*^2 D'^* + \frac{Ra_D}{Ra_M} u_z^*$$

$$\frac{dM'^*}{dt^*} = \frac{1}{\sqrt{Pr Ra_M}}\nabla_*^2 M'^* + u_z^*$$

3 Parameters in the equations

- Dry Rayleigh number Ra_D
- Moist Rayleigh number Ra_M
- Prandtl number Pr

And 2 are hidden in the buoyancy term:

$$B^* = \max\left(M'^*, D'^* + SSD + \left(1 - \frac{Ra_D}{Ra_M}\right) z^* - CSAz^*\right)$$

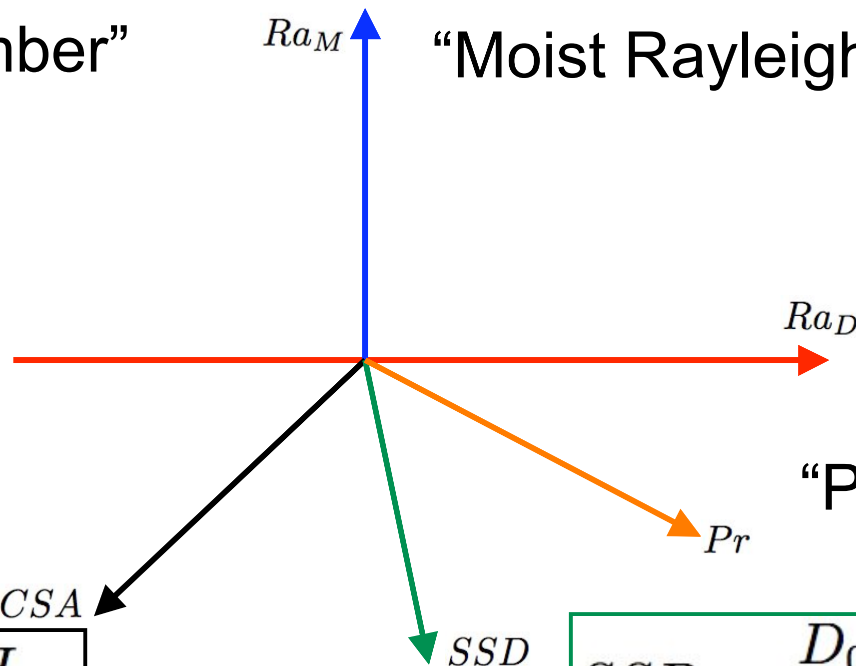
Five-dimensional parameter space

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

"Dry Rayleigh number"

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

"Moist Rayleigh number"



$$Pr = \frac{\nu}{\kappa}$$

"Prandtl number"

$$CSA = \frac{N_s^2 H}{M_0 - M_H}$$

"Condensation in Saturated Ascent"

$$SSD = \frac{D_0 - M_0}{M_0 - M_H}$$

"Surface Saturation Deficit"

2 Limiting cases:

Unsaturated atmosphere: if the whole atmosphere is unsaturated - i.e when

$$M_0 - D_0 - N_s^2 H \leq 0 \text{ and } M_H - D_H - N_s^2 H \leq 0$$

This problem is equivalent to the Rayleigh-Benard problem with $Ra = Ra_D$ and $\Pr = \Pr$

Saturated atmosphere: if the whole atmosphere is unsaturated - i.e when

$$M_0 - D_0 \geq 0 \text{ and } M_H - D_H \geq 0$$

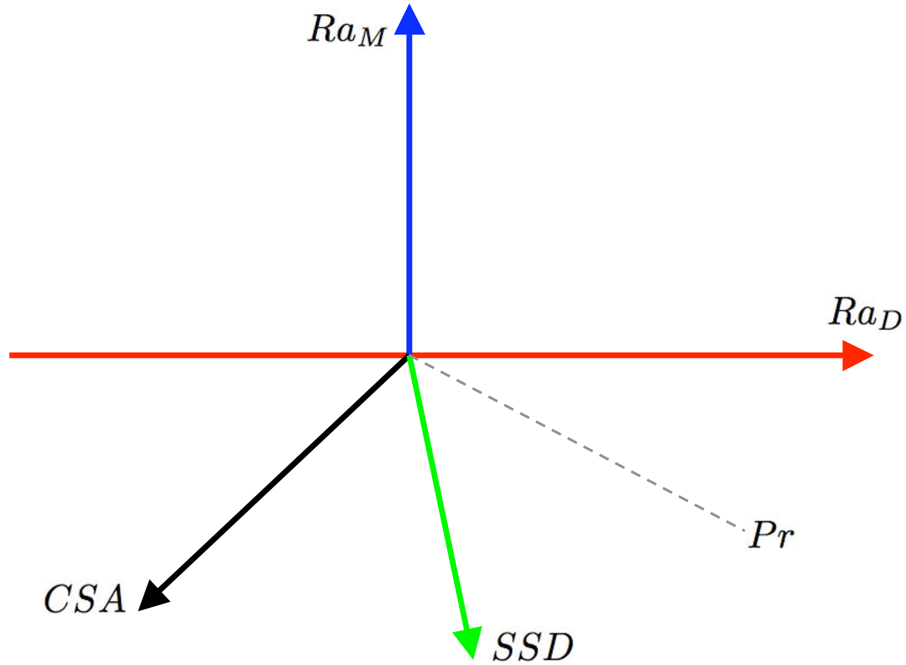
This problem is equivalent to the Rayleigh-Benard problem with $Ra = Ra_M$ and $\Pr = \Pr$

Atmospheric moist convection

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

$$Pr = 0.7$$

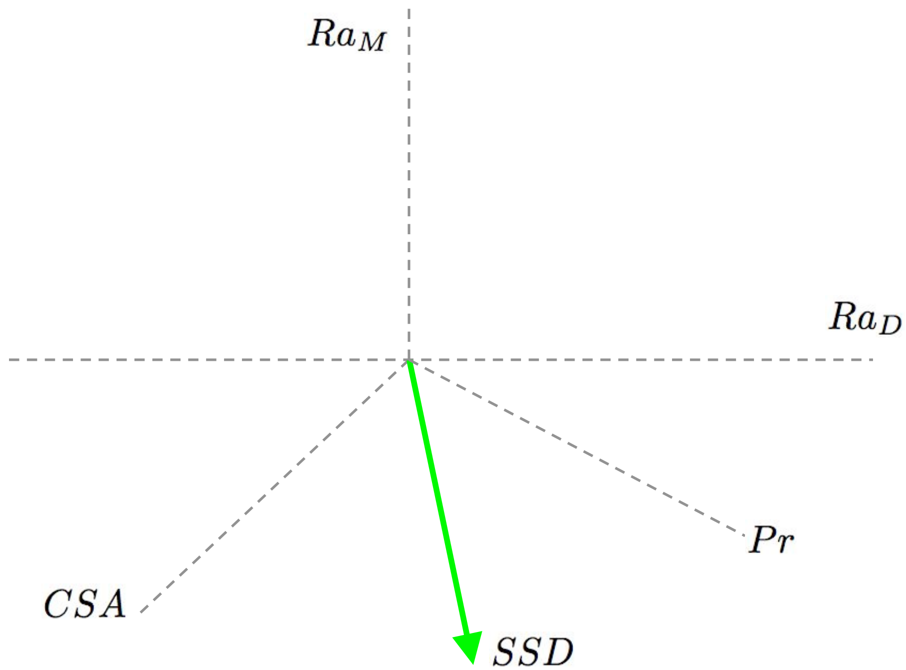


$$CSA = \frac{N_s^2 H}{M_0 - M_H}$$

$$SSD = \frac{D_0 - M_0}{M_0 - M_H}$$

Case 1: „stratocumulus regime“

$$Ra_D > 0 \quad (= const.) \qquad Ra_M > 0 \quad (= const.) \qquad Pr = 0.7$$



$$CSA = const.$$

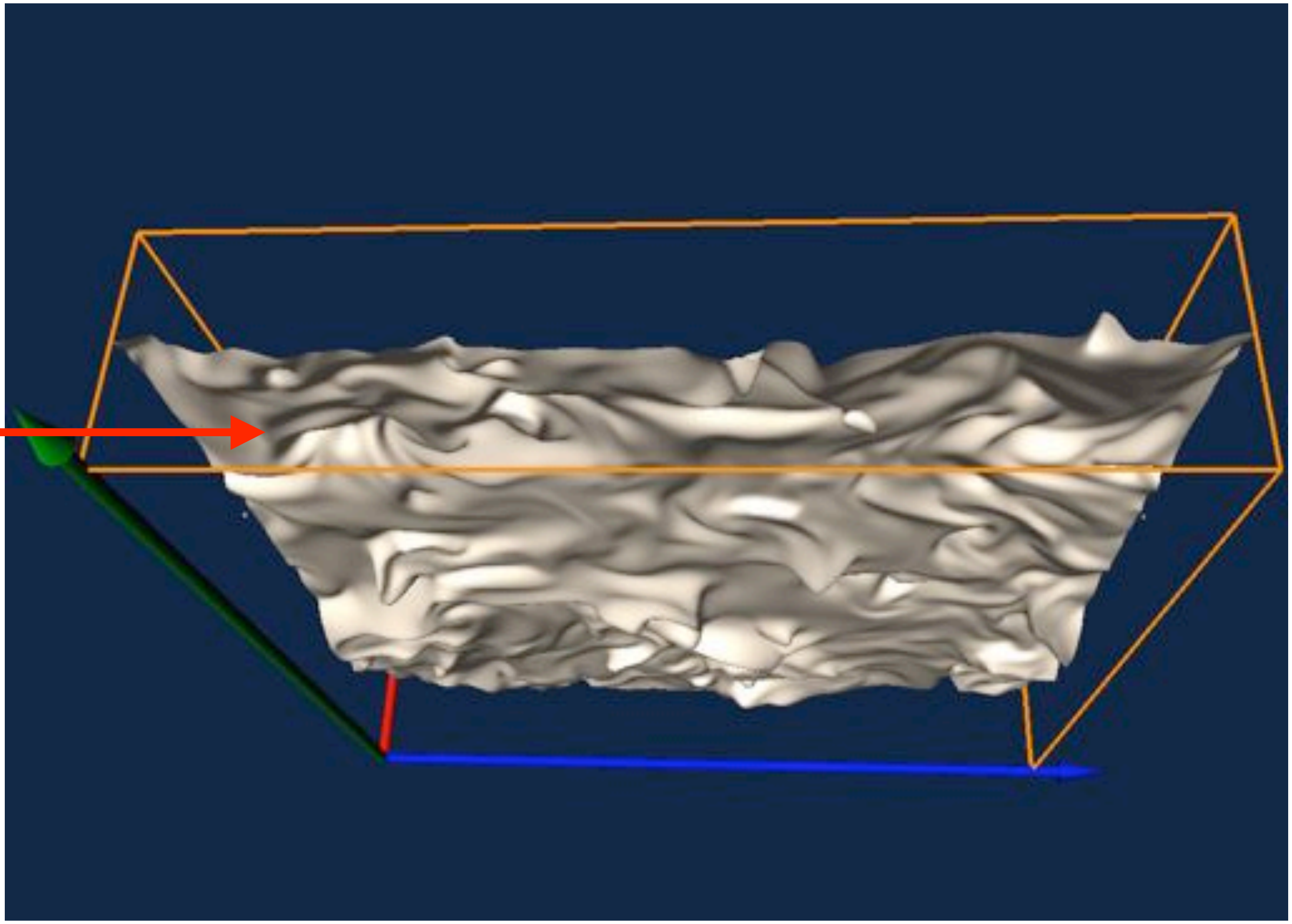
$$\boxed{SSD = \frac{D_0 - M_0}{M_0 - M_H}}$$

Buoyancy flux and cloud base

"Clouds" $$q_l \sim M - D + N_s^2 z \geq 0$$

"Cloud base"

$$M - D + N_s^2 z = 0$$



Buoyancy flux and cloud base

In Boussinesq system, generation of kinetic energy is given by the Integral of buoyancy flux:

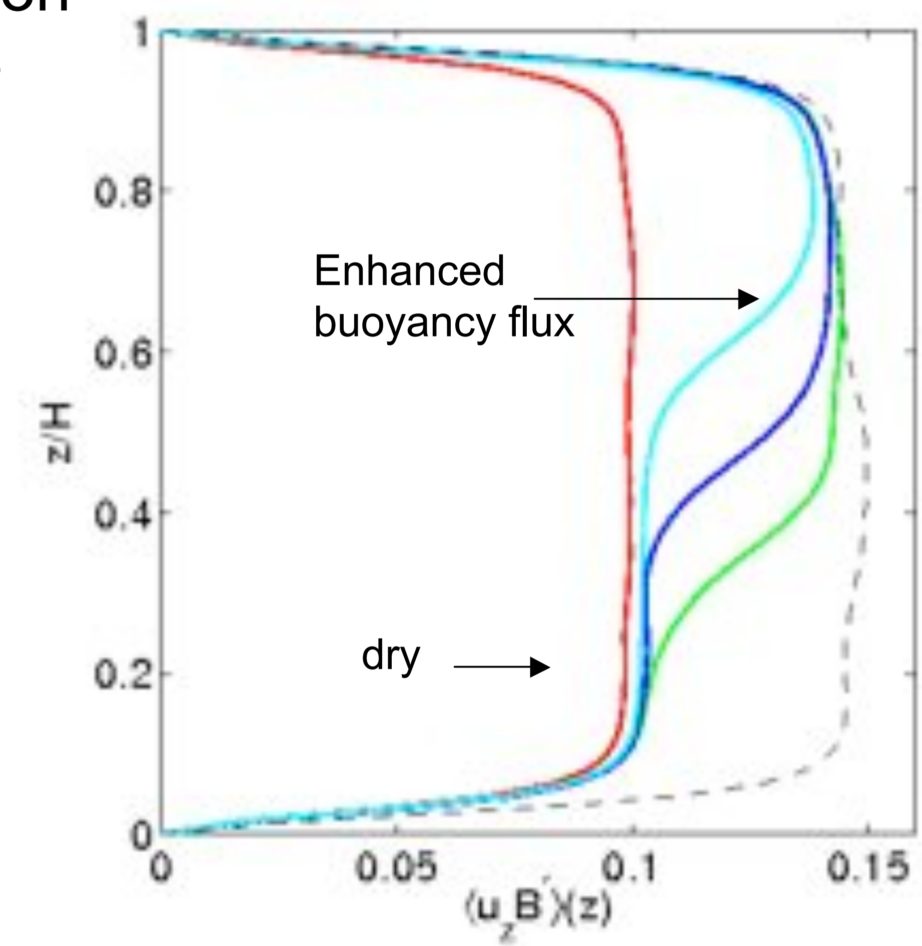
$$\partial_t KE = \int \overline{wB} dz - D$$

In classic Rayleigh-Benard convection, we have

$$\partial_t \overline{B} + \partial_z \overline{wB} = \kappa \partial_{zz} \overline{B}$$

so that the buoyancy flux is constant with height

But it is not the case for stratiform convection



Buoyancy flux and cloud base

$$B = \max(M, D - N_s^2 z)$$ (not an adiabatic invariant anymore!)

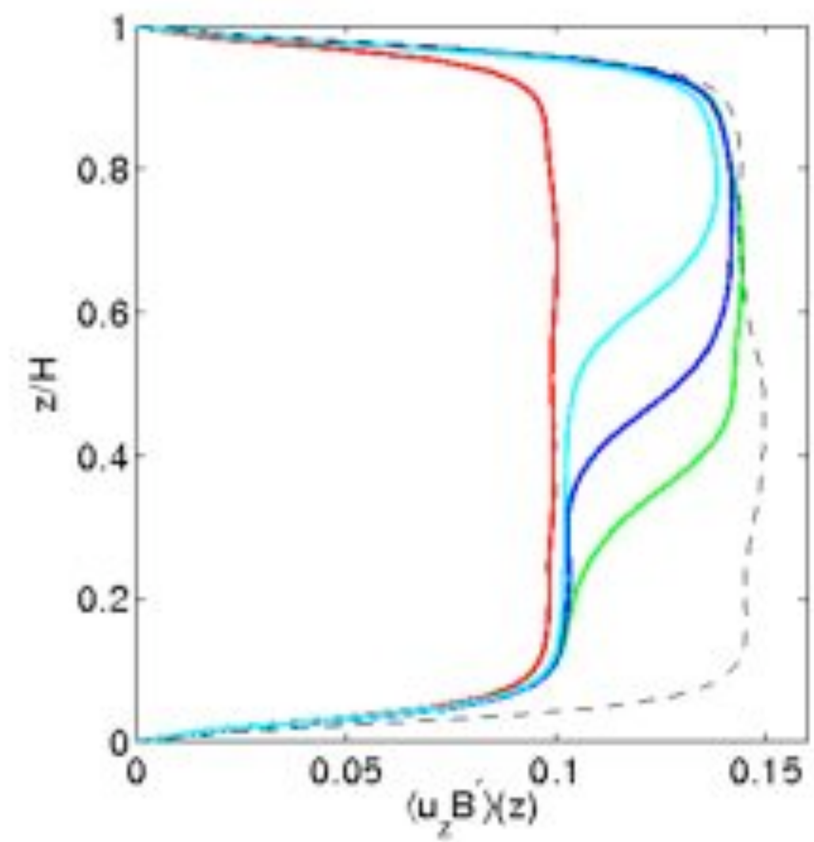
$$\partial_t \overline{D} + \partial_z \overline{wD} = \kappa \partial_{zz} \overline{D}$$
$$\partial_t \overline{M} + \partial_z \overline{wM} = \kappa \partial_{zz} \overline{M}$$

In unsaturated regions:

$$\overline{wB} = \overline{wD}$$

In saturated regions:

$$\overline{wB} = \overline{wM}$$



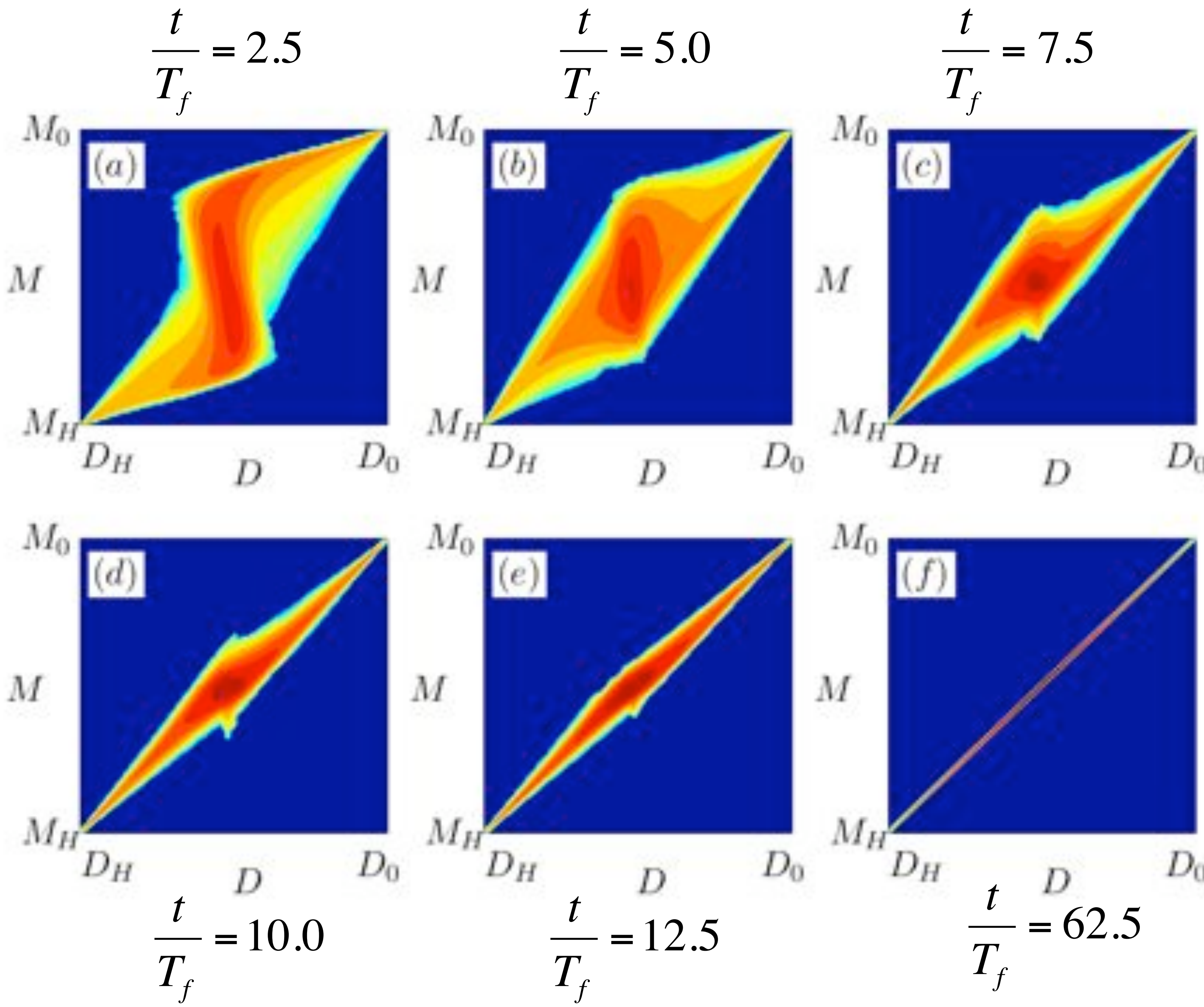
Cloud base of stratocumulus and water deficit increase

Mixing line

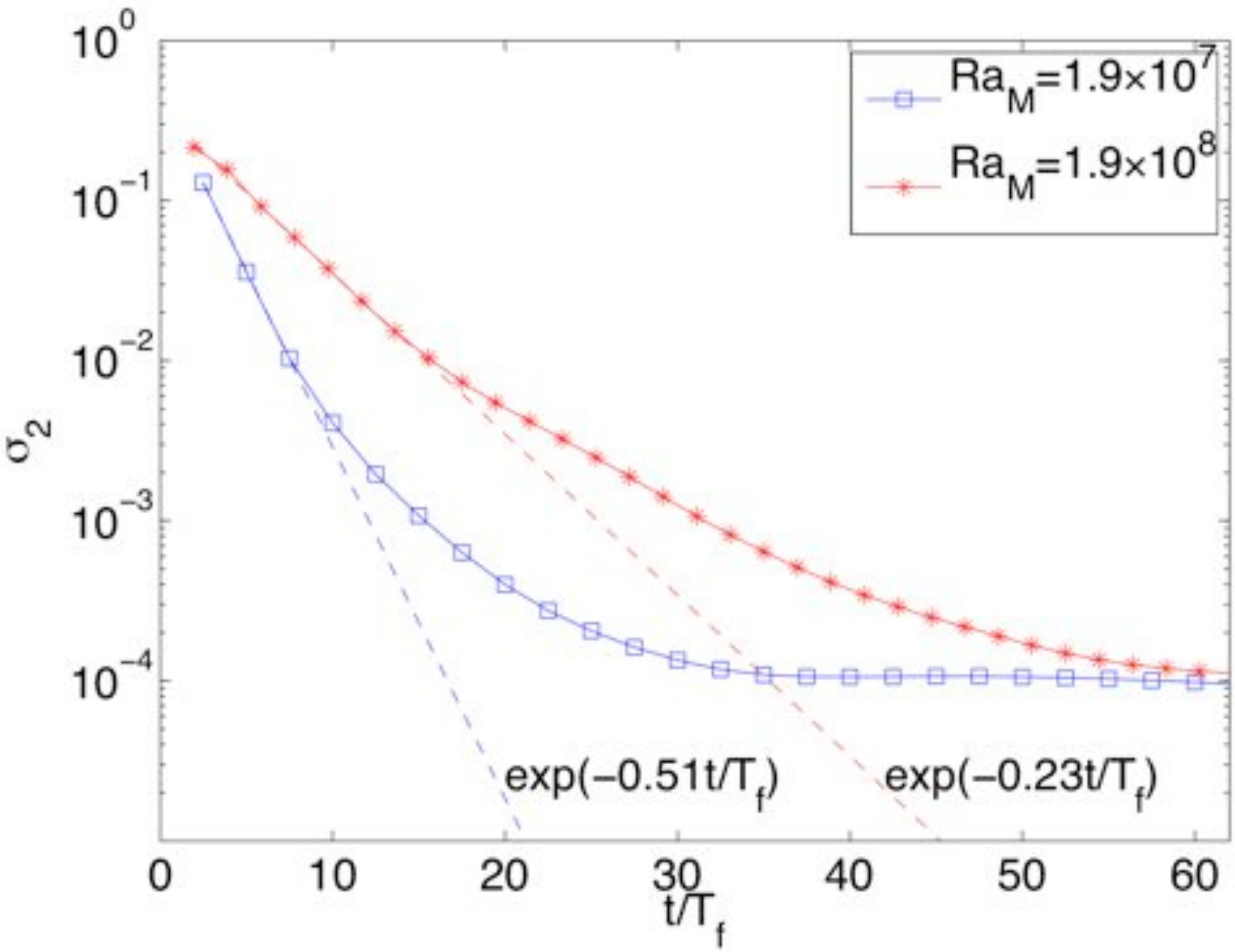
- Over long time-scale, the solution collapse toward a mixing line, I.e. the dry and moist buoyancy can be expressed as a function of a mixing fraction

$$M = \chi M_H + (1-\chi) M_0$$

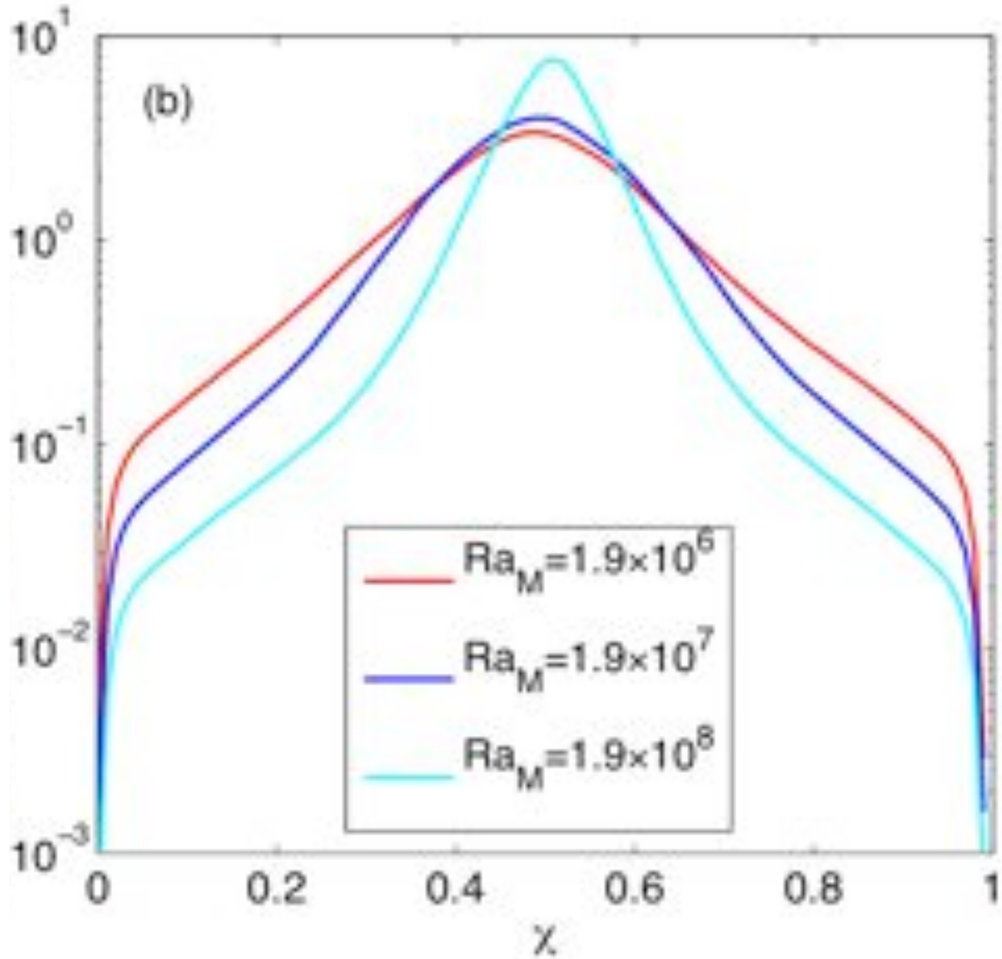
$$D = \chi D_H + (1-\chi) D_0$$



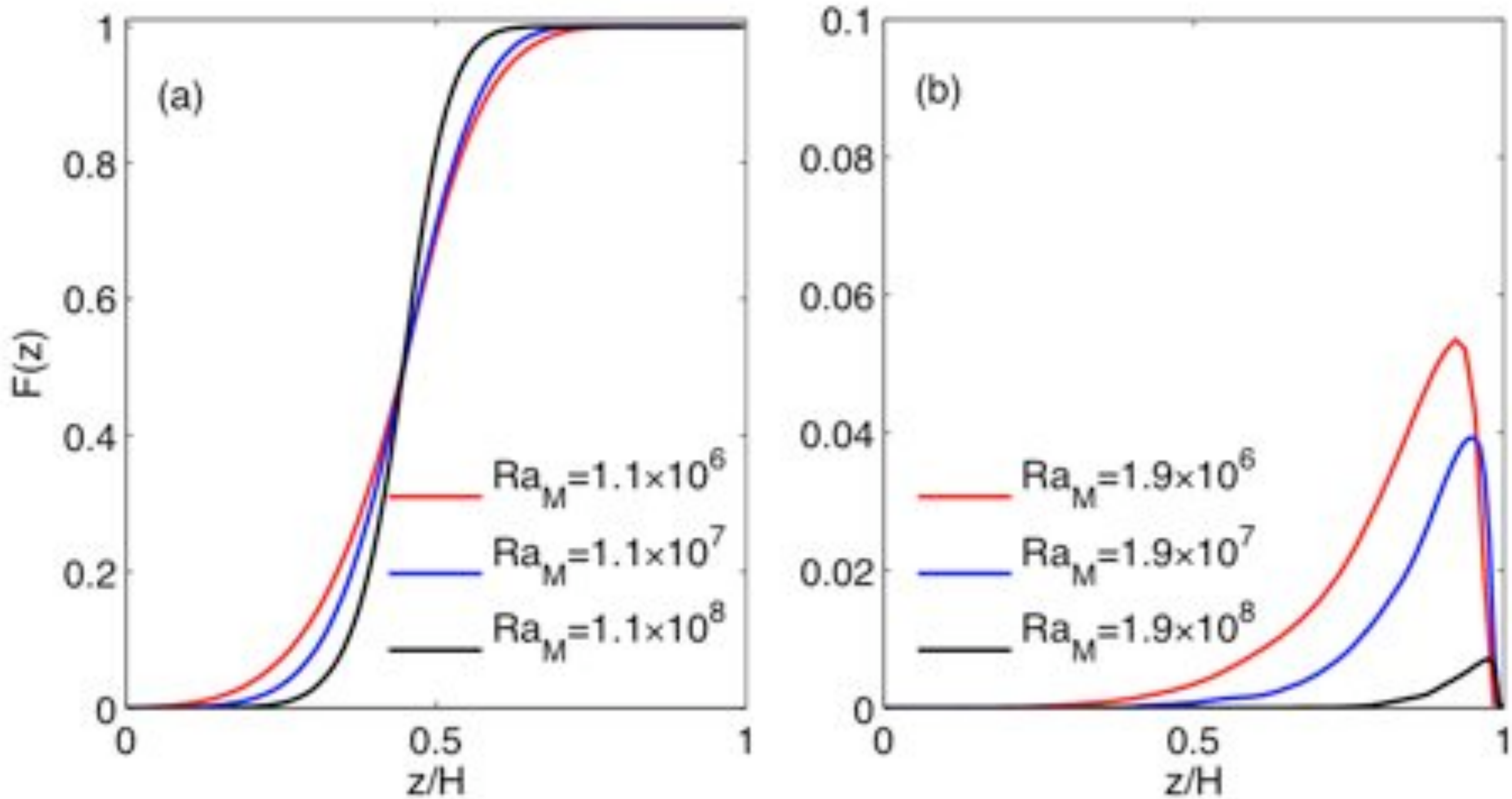
$\frac{t}{T_f} = 2.5$
$\frac{t}{T_f} = 5.0$
$\frac{t}{T_f} = 7.5$
(a)
(b)
(c)
M_0
M
M_H
D_H
D
D_0
(d)
(e)
(f)
$\frac{t}{T_f} = 10.0$
$\frac{t}{T_f} = 12.5$
$\frac{t}{T_f} = 62.5$



- However, rate of collapse decrease with Rayleigh number



- But the steady state distribution depends also on the Rayleigh number



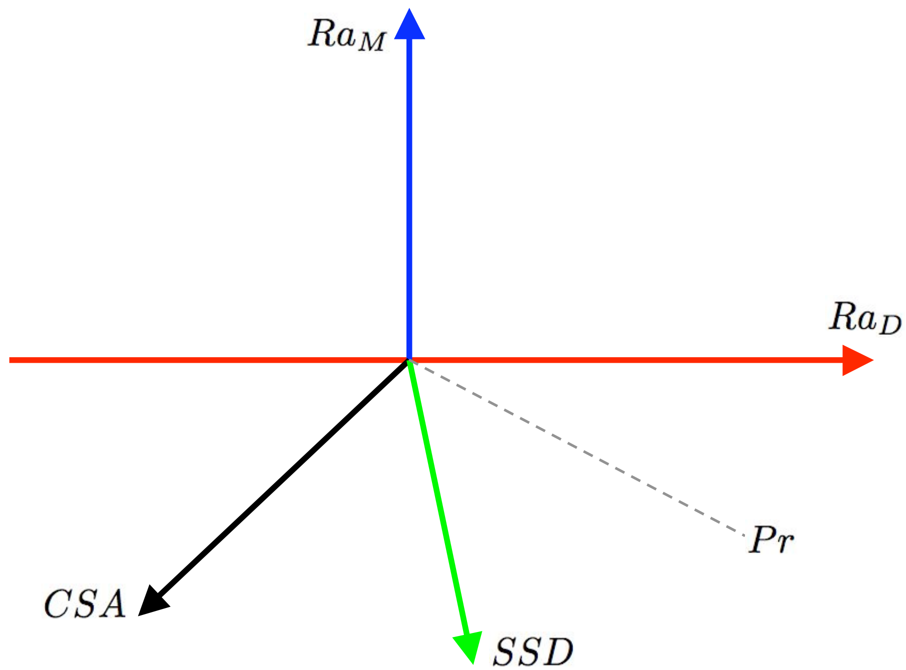
- Which has direct impact on the cloudbase/cloud fraction

Case 2: stratocumulus to cumulus

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

$$Pr = 0.7$$



$$CSA = \frac{N_s^2 H}{M_0 - M_H}$$

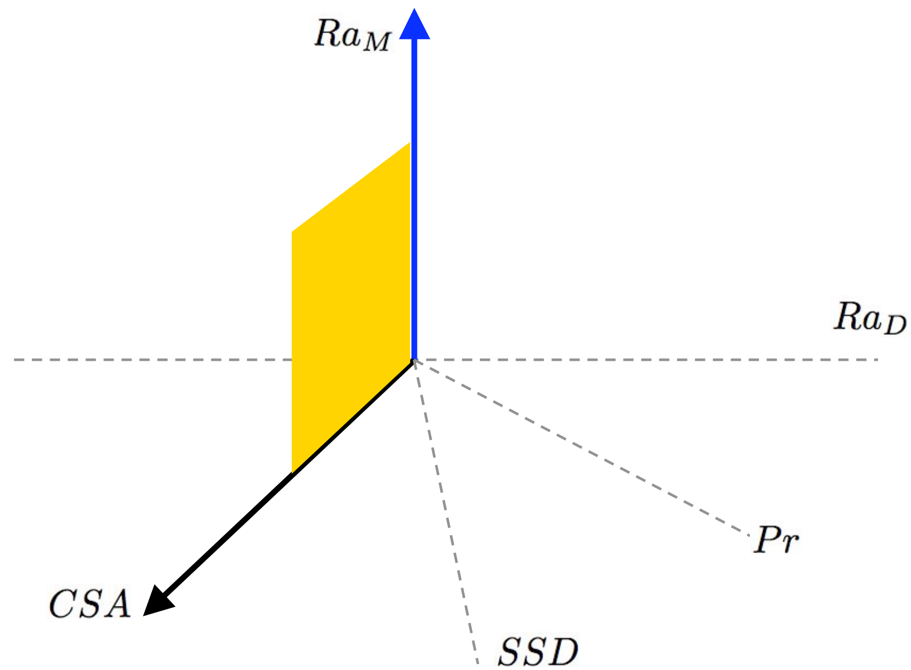
$$SSD = \frac{D_0 - M_0}{M_0 - M_H}$$

Changing cloud fraction

$$Ra_D > 0 \quad (= const.)$$

$$\boxed{Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}}$$

$$Pr = 0.7$$

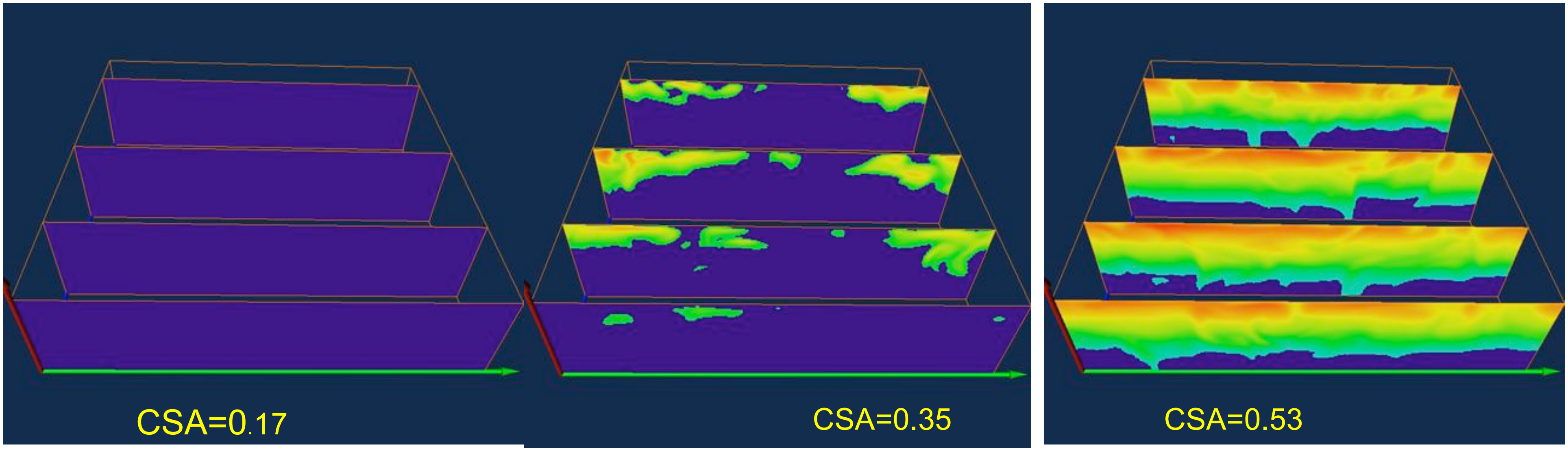


$$\boxed{CSA = \frac{N_s^2 H}{M_0 - M_H}}$$

$$SSD = 0$$

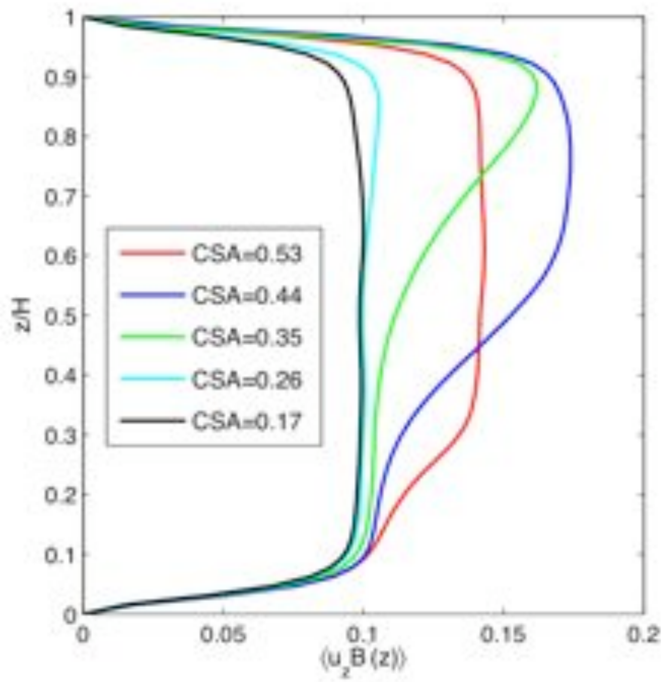
Variation of M_H and thus of CSA and Ra_M

„Stratocumulus“ to „Cumulus“



Transition to „Cumulus“

Cloud layer breaks up and disappears

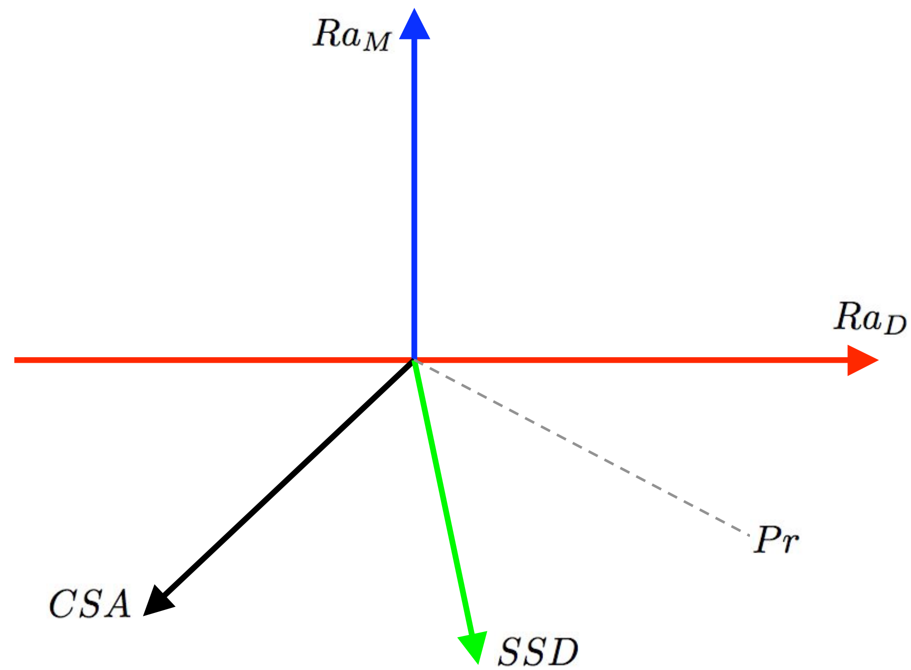


Case 3: Conditional instability

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

$$Pr = 0.7$$



$$CSA = \frac{N_s^2 H}{M_0 - M_H}$$

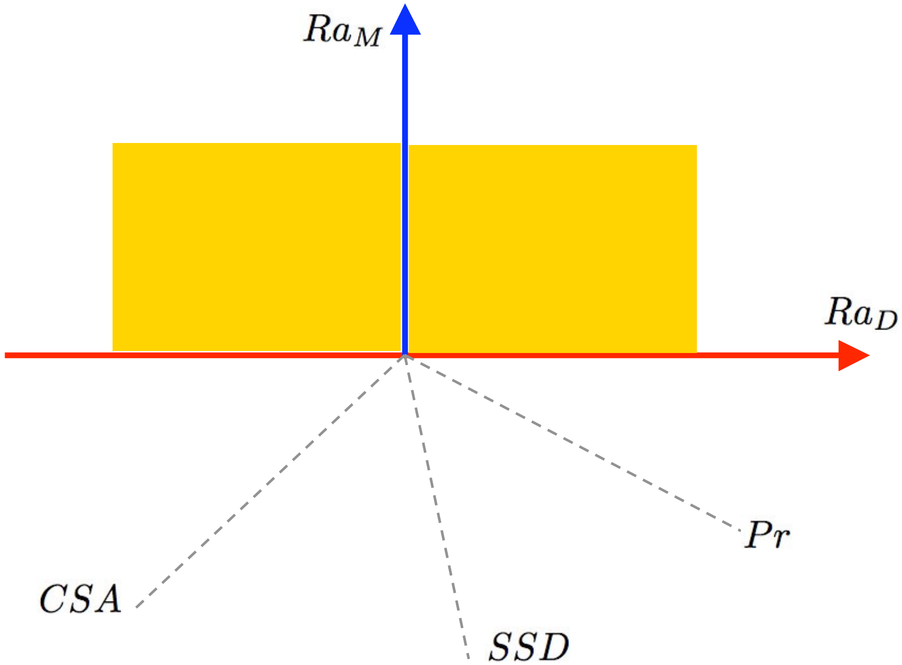
$$SSD = \frac{D_0 - M_0}{M_0 - M_H}$$

Conditional instability

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

$$Pr = 0.7$$



$$CSA > 0 \quad (= const.)$$

$$SSD = 0$$

Conditional instability

$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

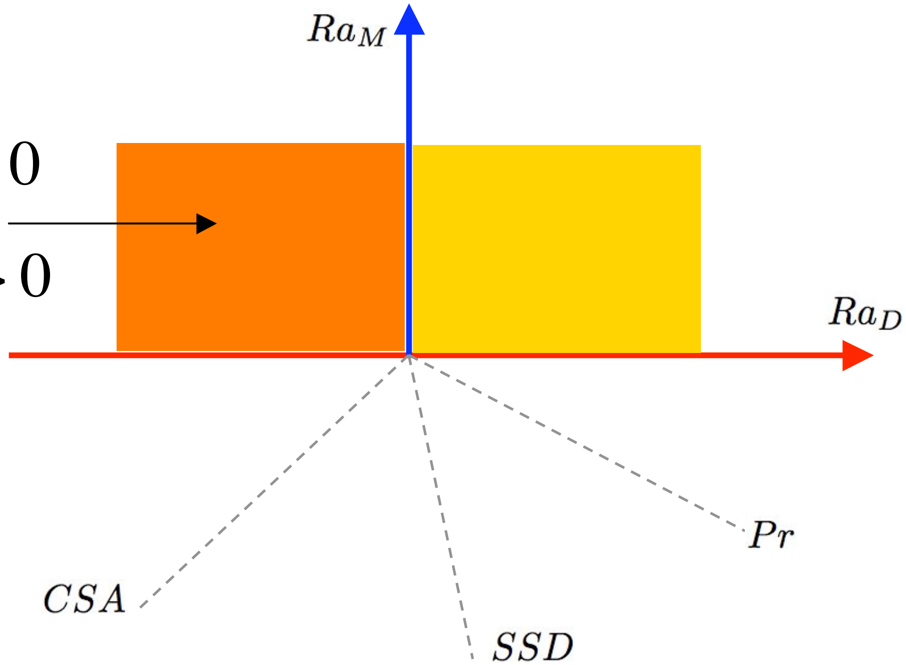
$$Pr = 0.7$$

Stable stratification for unsaturated parcels

$Ra_D < 0$

$Ra_M > 0$

But unstable stratification for saturated parcels



$CSA > 0 \quad (= const.)$

$SSD = 0$

Conditional instability

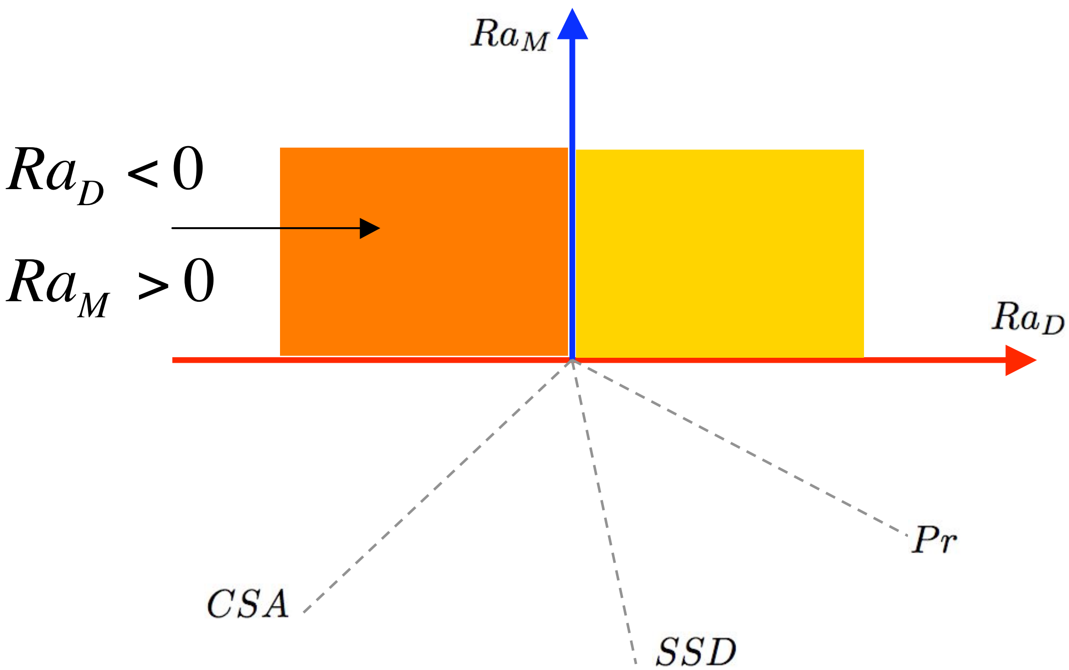
$$Ra_D = \frac{(D_0 - D_H)H^3}{\nu\kappa}$$

$$Ra_M = \frac{(M_0 - M_H)H^3}{\nu\kappa}$$

$$Pr = 0.7$$

Stable stratification for unsaturated parcels

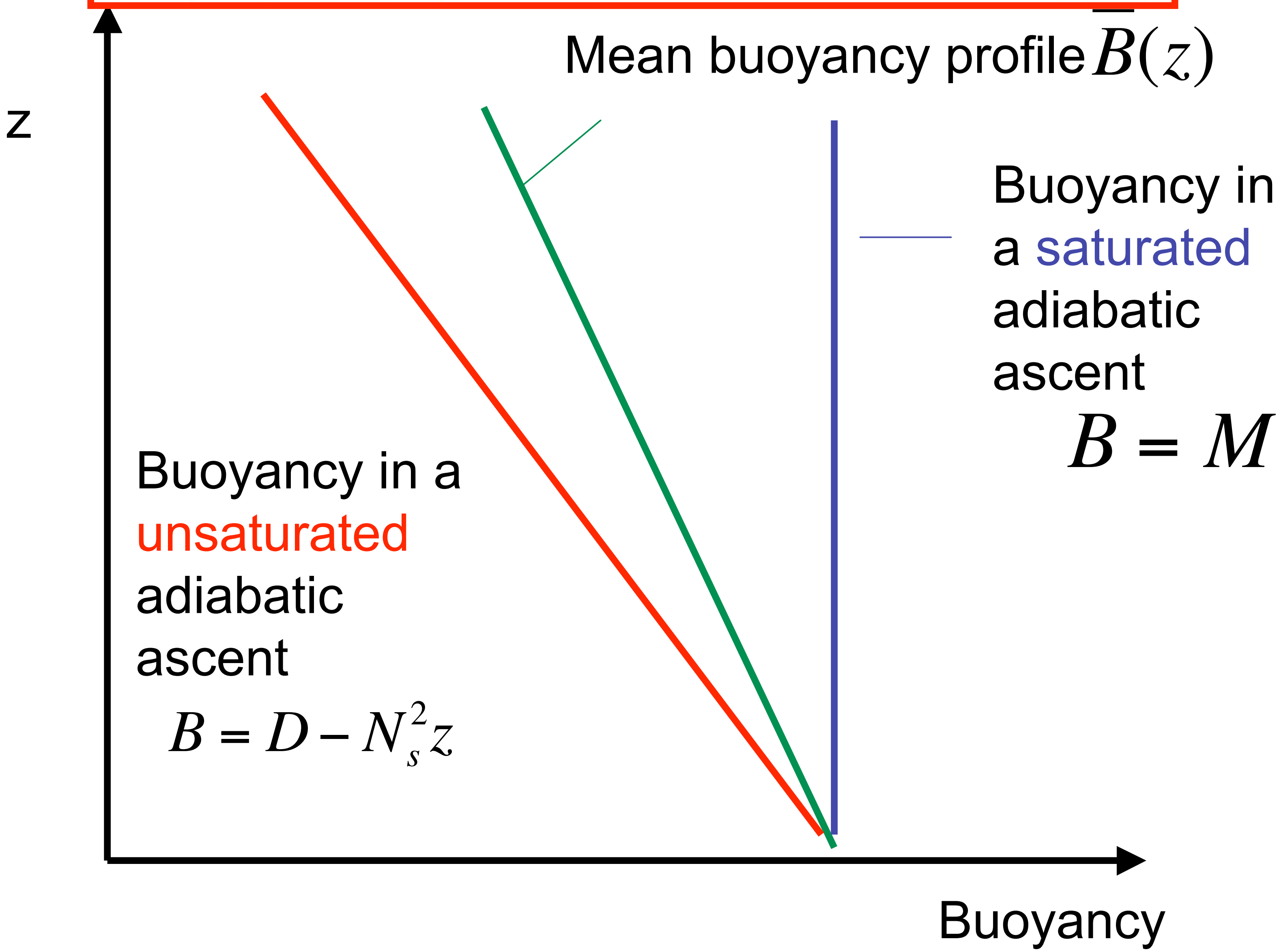
But unstable stratification for saturated parcels

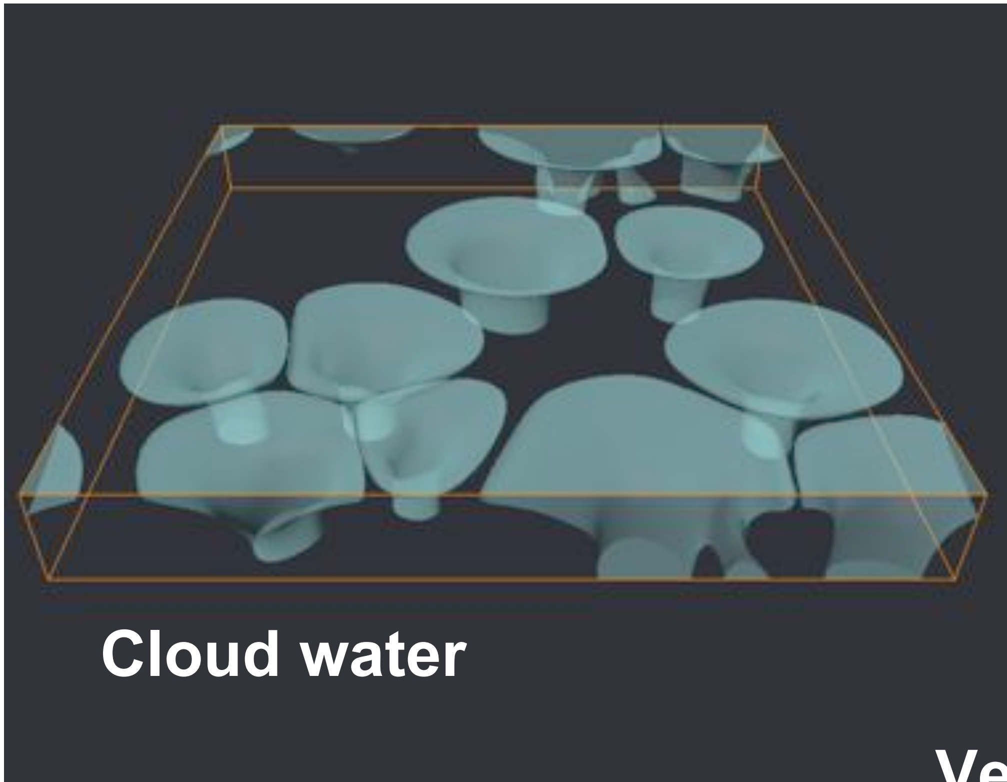


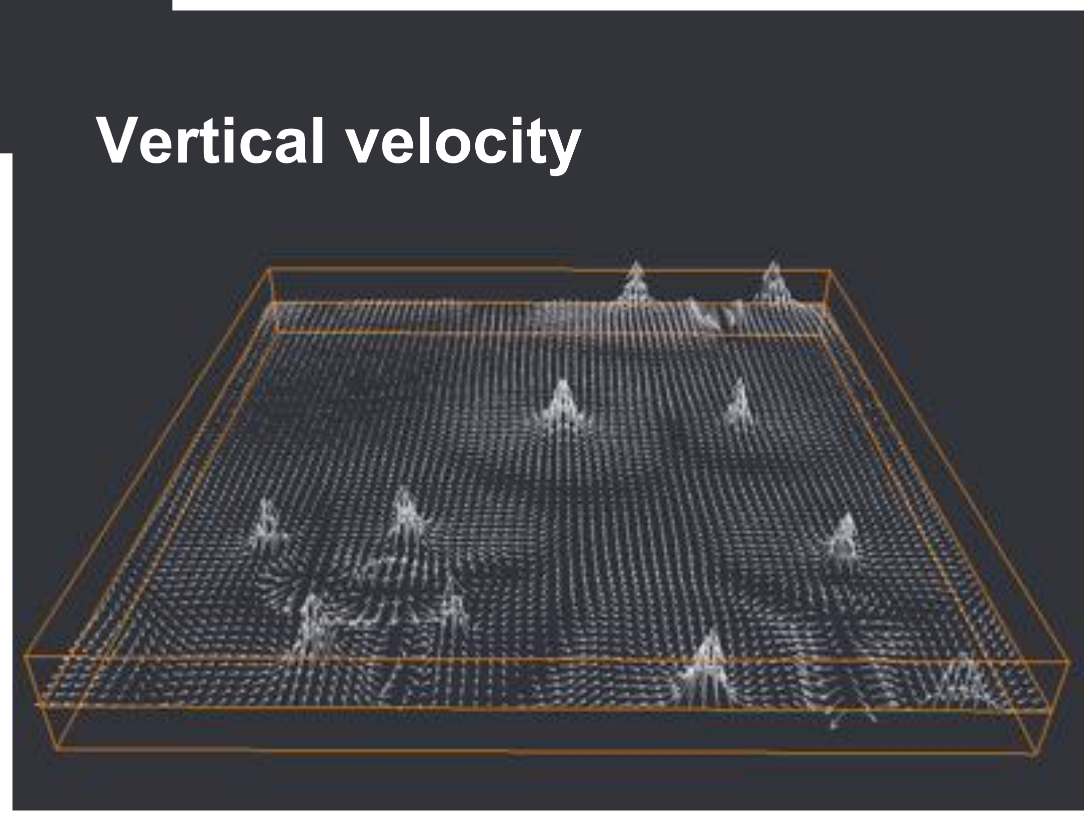
$$CSA > 0 \quad (= const.)$$

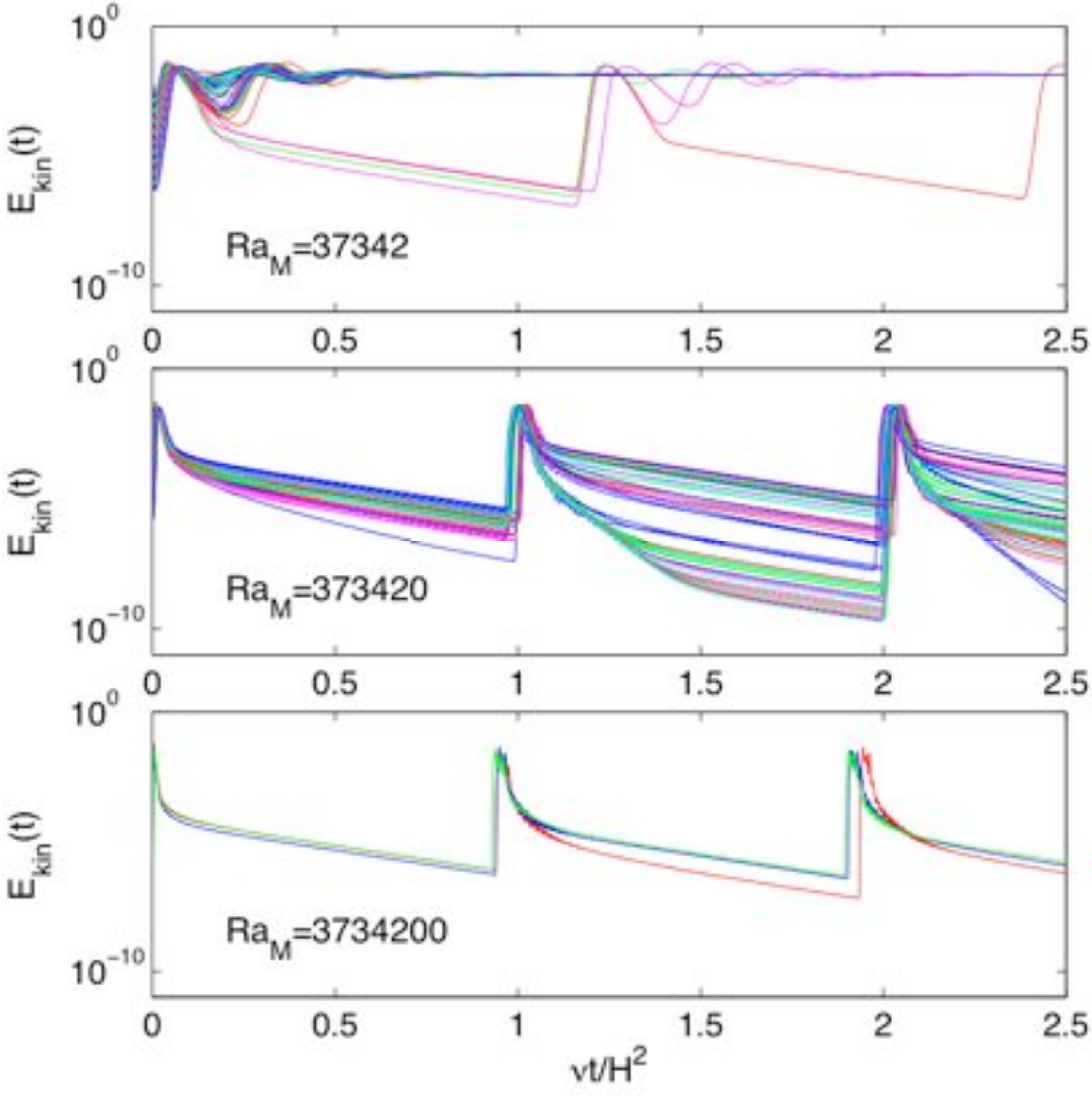
$$SSD = 0$$

$$\boxed{B(D, M, z) = \max(M, D - N_s^2 z)}$$



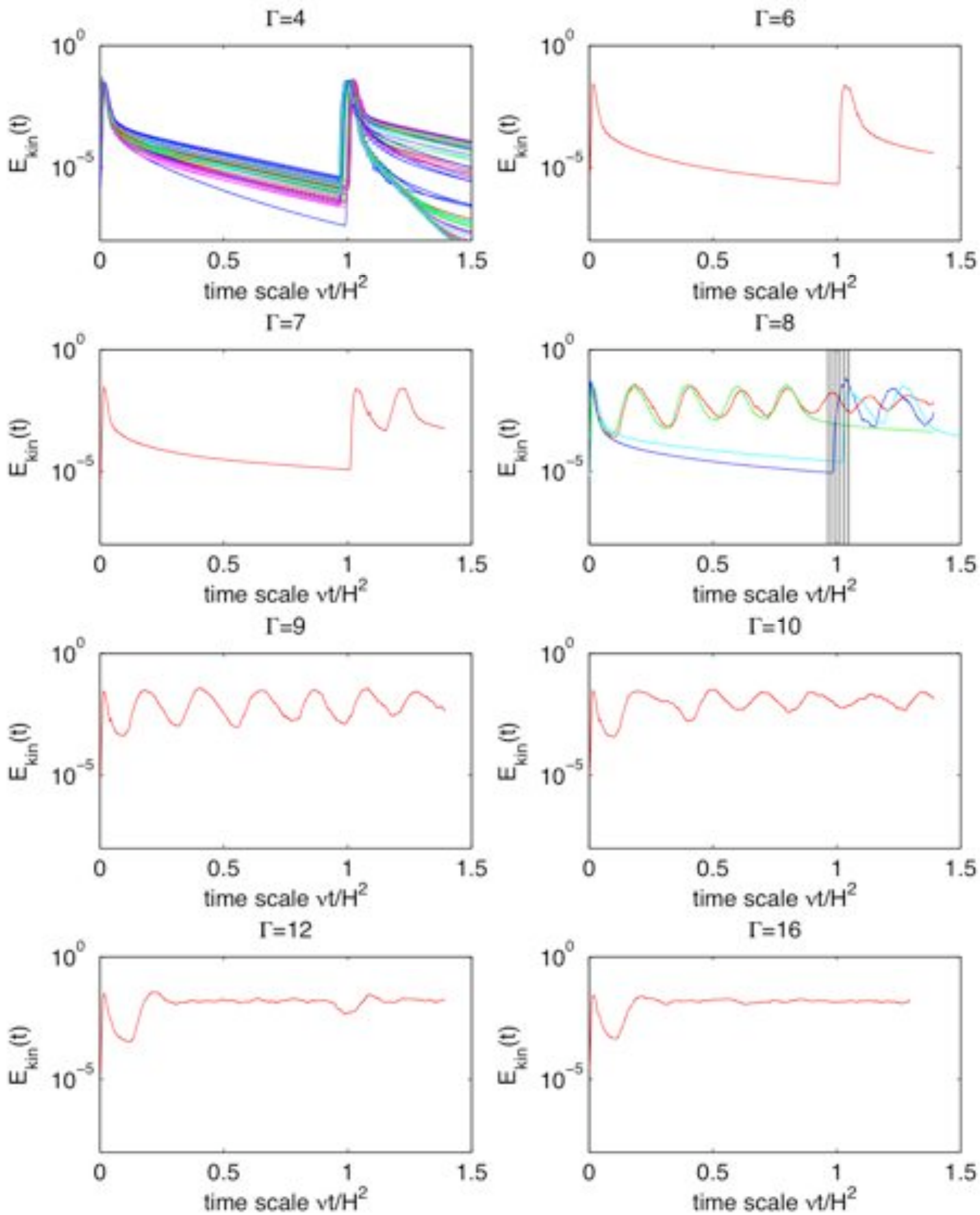

Cloud water

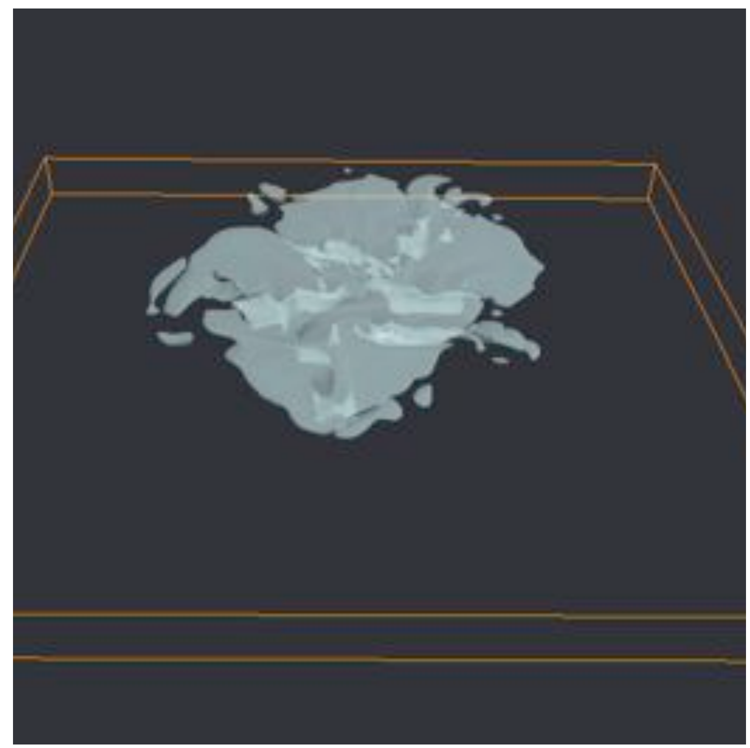

Vertical velocity



And convection becomes inceasingly intermittent at high Ra.

Simulations evolve toward a localized turbulent patch at high aspect ratio





Conclusion

- Atmosphere can be viewed as a heat engine that generates kinetic energy by transporting energy from warm to cold.
- Relative humidity is a key factor in determining how much work is produced by atmospheric circulation. This can be captured by a simple steam cycle.
- This behavior is related to the non-linear state equation associated with the different behavior between staurated and unsaturated air.
- A simplified piecewise linear equation of state can capture the main effect of phase transition on dynamics, and use to simulat idealized moist convection.

- Latent and sensible heat flux:

$$Q_{lat} = L\Delta r_T$$

$$Q_{sen} = T_{in}\Delta S + (g_{in} - L)\Delta r_T$$

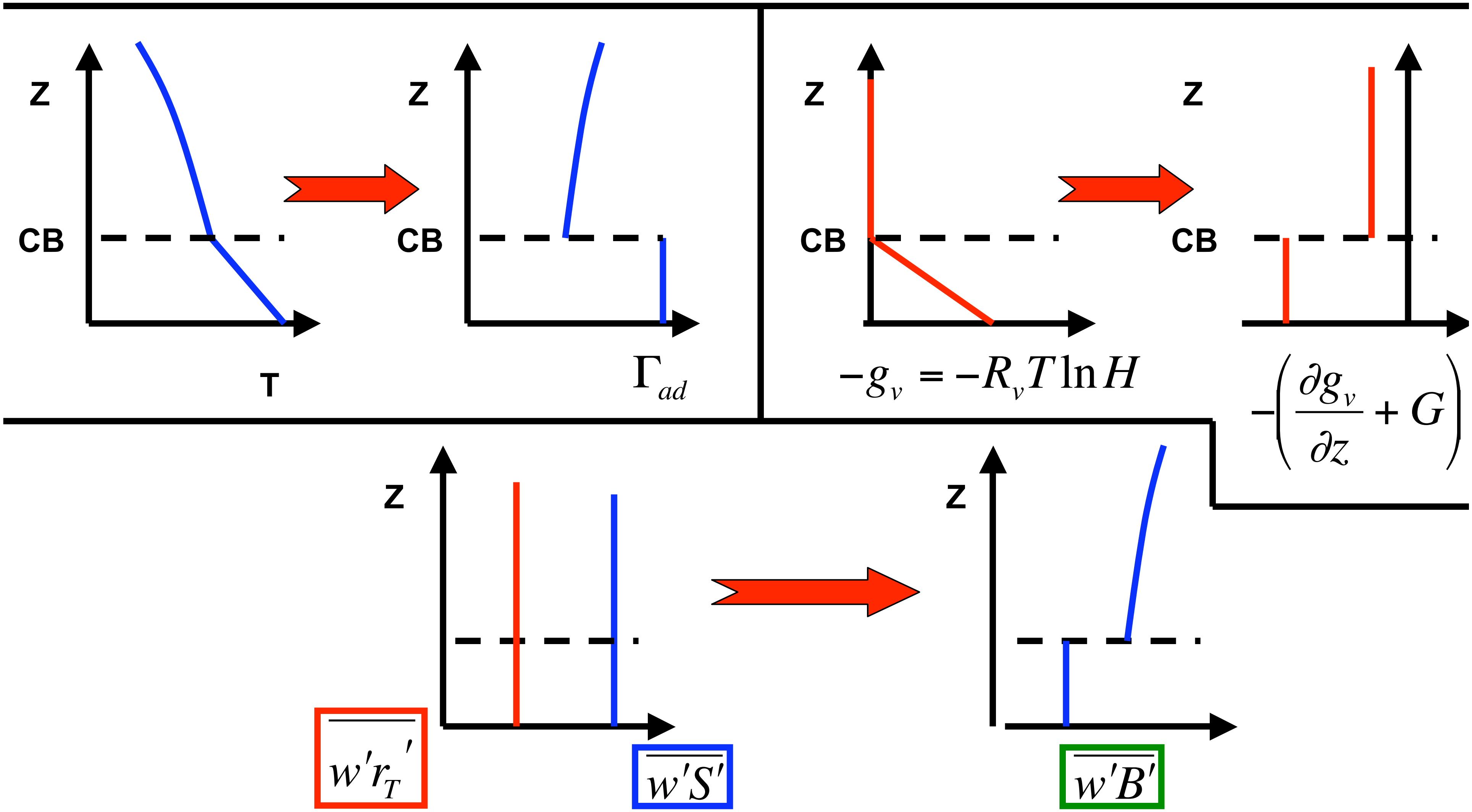
- Bowen ratio:

$$B = \frac{Q_{sen}}{Q_{lat}}$$

$$\boxed{\begin{aligned}\text{Efficieny } \eta &= \frac{B}{1+B}\eta_c + \frac{1}{1+B}\eta_H \\ &= \frac{T_{in} - T_{out}}{T_{in}} + \frac{1}{1+B}\frac{R_v T_{out}}{L}\ln\frac{H_{in}}{H_{out}}\end{aligned}}$$

The efficiency of a mixed cycle depends on both relative humidity and Bowen ratio.

$$\rho_0 \overline{w'B'} \cong \Gamma_{ad} \rho_0 \overline{w'S'} - \left(\frac{\partial g_v}{\partial z} + G \right) \rho_0 \overline{w' r_T{}'}$$



II. Buoyancy flux

- We consider now a convective layer. The generation of kinetic energy is given by the vertical integral of the buoyancy flux:

$$\frac{dKE}{dt} = \int \rho_0 \overline{w'B'} dz$$

- B is the buoyancy

$$B(S, r_T, z) = G \frac{\alpha_p - \alpha_0(z)}{\alpha_0(z)}$$

A. Stratocumulus convection

- Linearize the buoyancy flux

$$\boxed{\rho_0 \overline{w'B'} \cong \left(\frac{\partial B}{\partial S}\right)_{r_T,p} \rho_0 \overline{w'S'} + \left(\frac{\partial B}{\partial r_T}\right)_{S,p} \rho_0 \overline{w'r_T{}'}}$$

- The partial derivatives can be rewritten using the Maxwell relationships:

$$\left(\frac{\partial B}{\partial S}\right)_{r_T,p} = \rho_0 G\left(\frac{\partial \alpha}{\partial S}\right)_{r_T,p} = \rho_0 G\left(\frac{\partial T}{\partial p}\right)_{r_T,S} = -\left(\frac{\partial T}{\partial z}\right)_{r_T,S} = \Gamma_{ad}$$

$$\left(\frac{\partial B}{\partial r_T}\right)_{S,p} = \rho_0 G\left(\frac{\partial \alpha_d}{\partial r_T}\right)_{S,p} - G = \rho_0 G\left(\frac{\partial g_v}{\partial p}\right)_{r_T,S} - G = -\left(\frac{\partial g_v}{\partial z}\right)_{r_T,S} - G$$

- After integration:

$$\frac{dKE}{dt} = \int_0^{\Delta z} \rho_0 \overline{w'B'} dz$$

$$= (T_{in} - T_{out}) \rho_0 \overline{w'S'} + \left(g_{v,in} - g_{v,out}\right) \rho_0 \overline{w' r_T{}'} - G \Delta z \rho_0 \overline{w' r_T{}'}$$

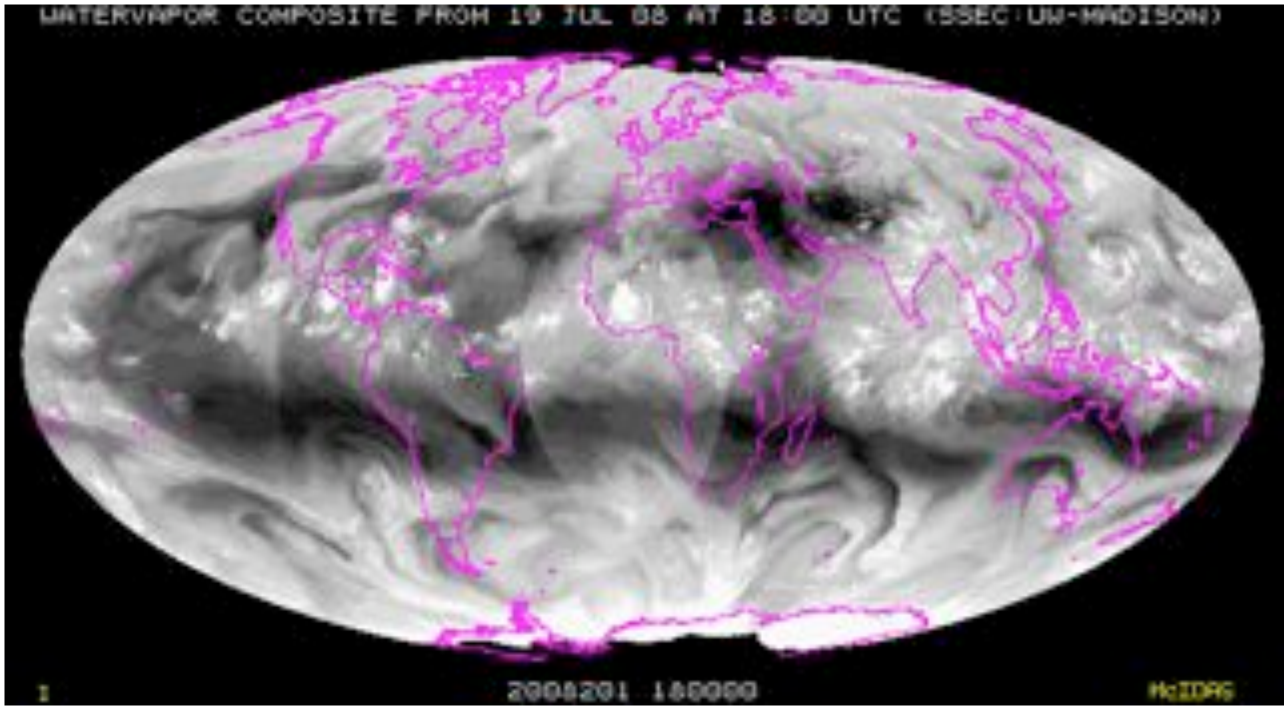
Work done by a Carnot cycle

“water vapor penalty”

geopotential energy gained by water

Work done by a mixed cycle

Thermodynamics of moist air
Part 2: Global cosiderations



Olivier Pauluis (NYU)

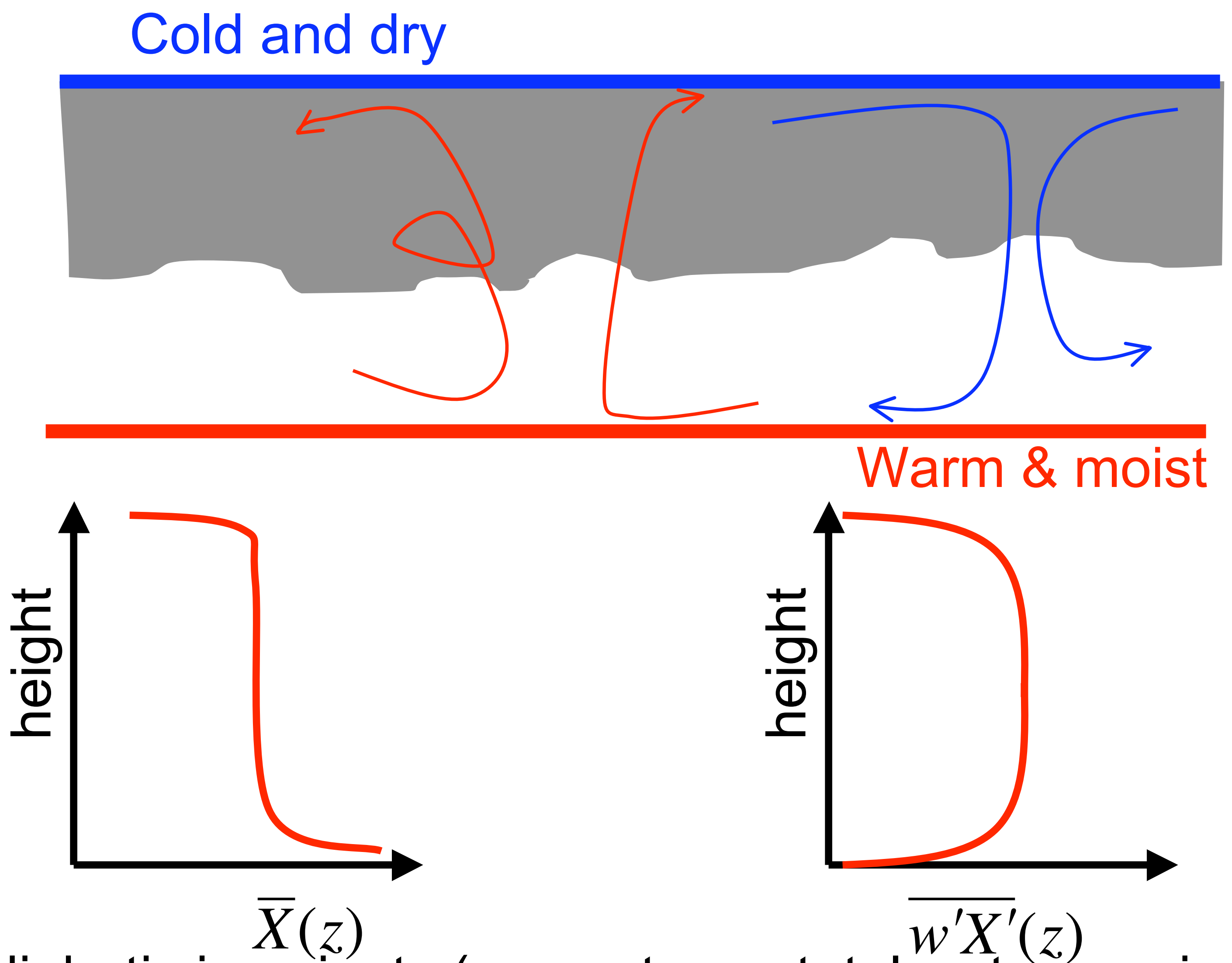
Porquerolles - Sept. 2010

Outline

- Introduction: the nature of the problem…
- Entropy of moist air
- Heat engines and water vapor.
- Equation of state of moist air
- Idealized moist Rayleigh-Benard convection

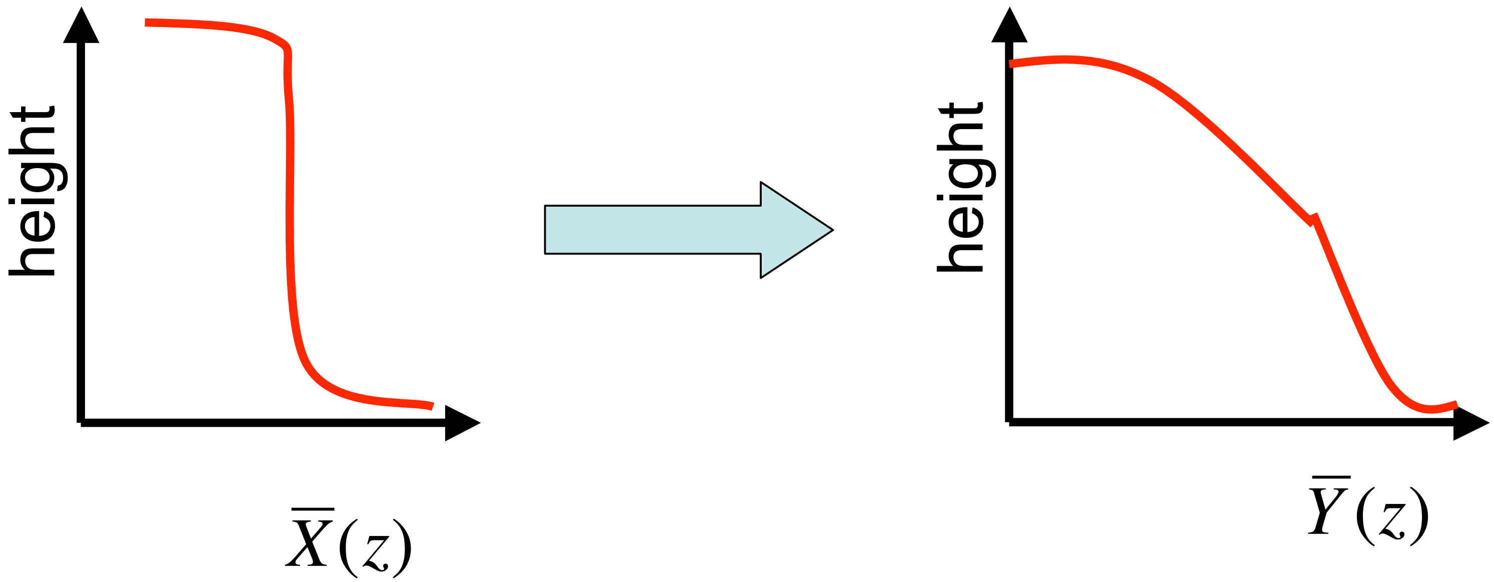
➔ Mixing in stratocumulus

- Buoyancy and entropy fluxes
- Isentropic circulation



Adiabatic invariants (e.g. entropy, total water, equivalent potential temperature, M and D…) are well-mixed.
And their flux is constant.

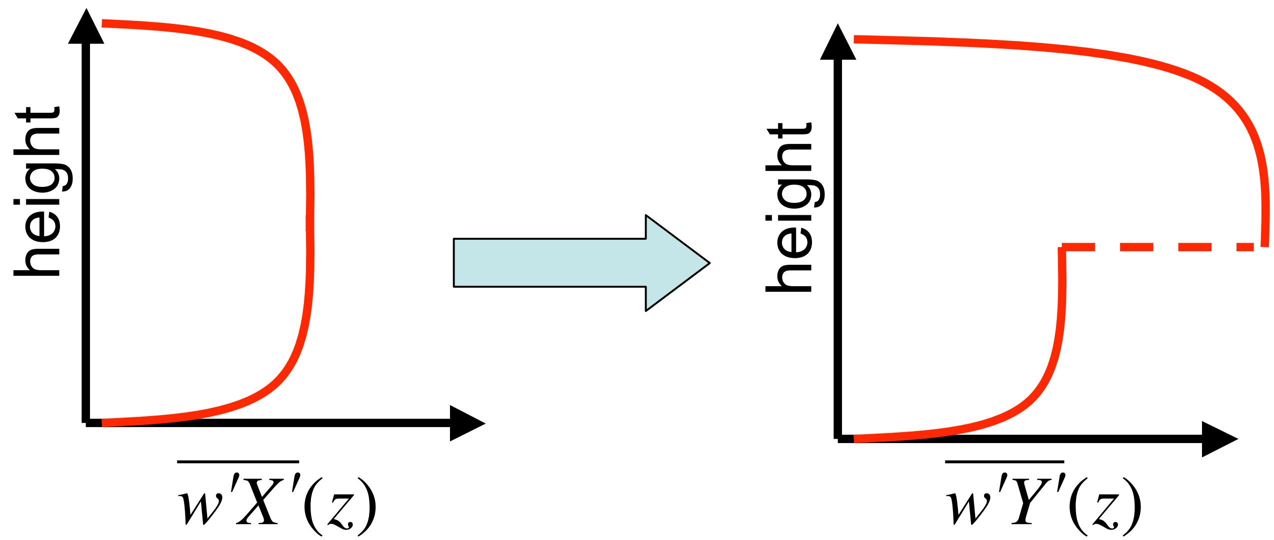
For non-invariants (e.g. liquid water, temperature, buoyancy)



$$\bar{Y}(z) = F(\bar{X}_1(z), \bar{X}_2(z), p(z))$$

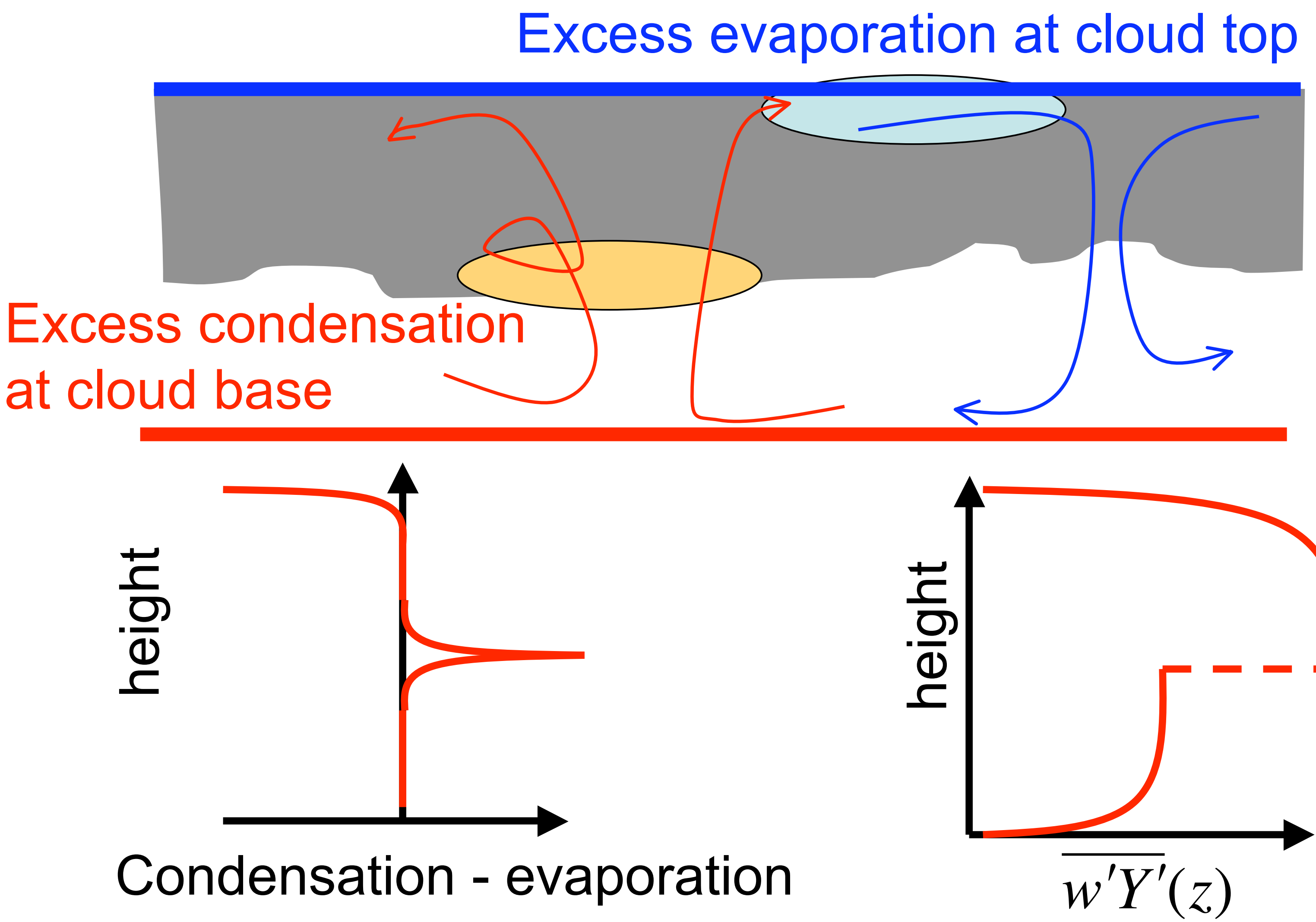
➔The vertical derivative changes abruptly at the cloud base

For non-invariants (e.g. liquid water, temperature, buoyancy)



$$\overline{w'Y'} = \frac{\partial Y}{\partial X_1}\overline{w'X_1{}'} + \frac{\partial Y}{\partial X_2}\overline{w'X_2{}'}$$

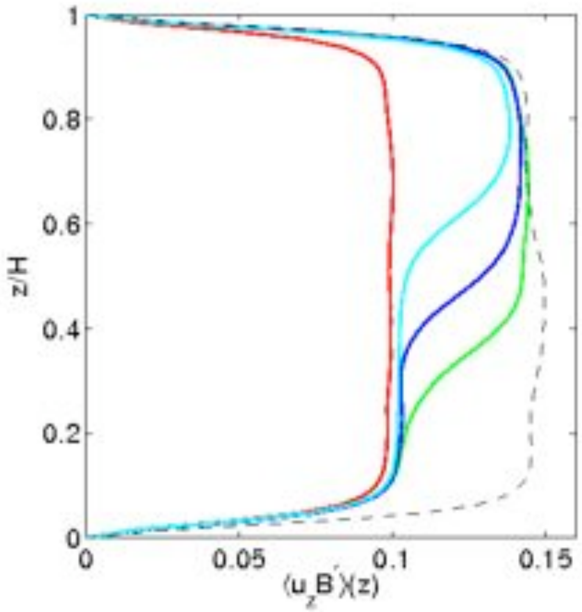
➔The vertical flux is discontinuous at cloud base!



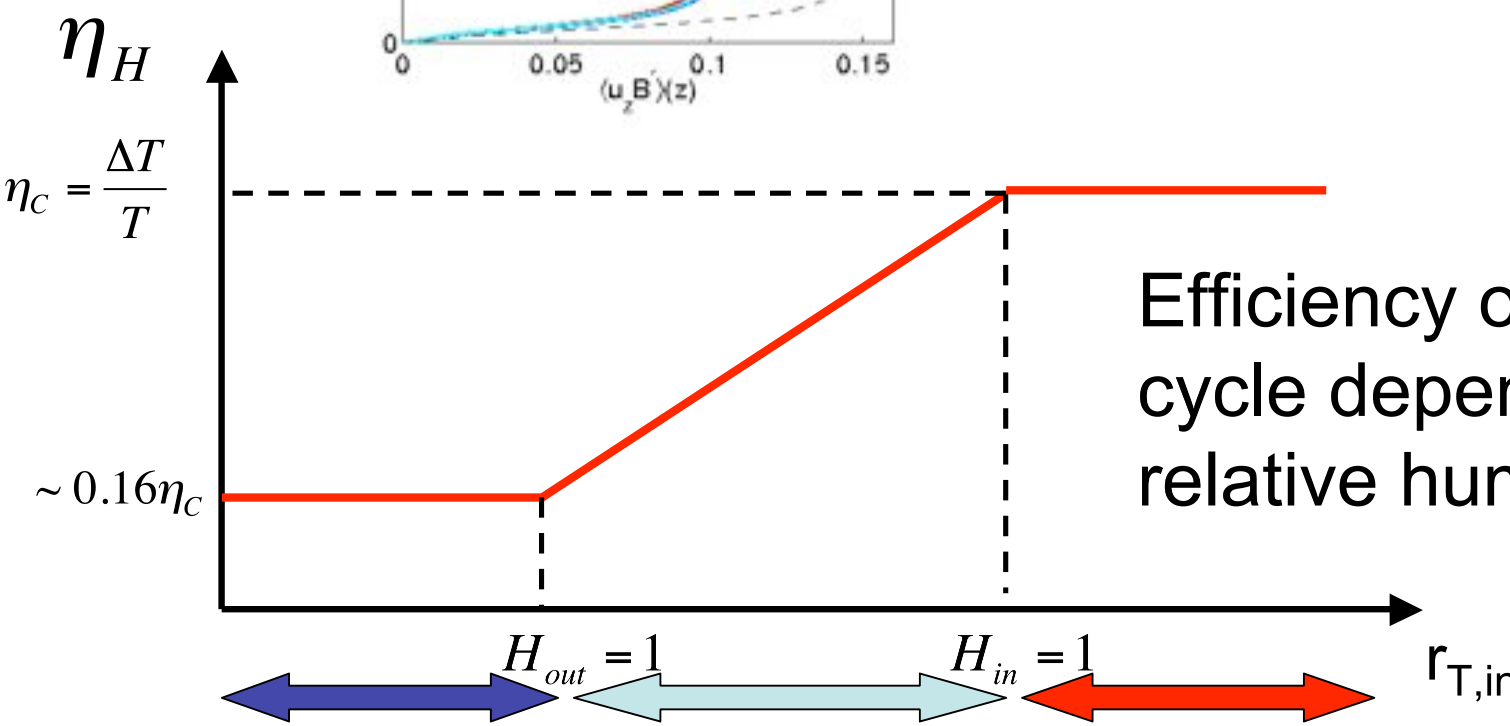
Excess condensation at cloud base acts as a source (or sink) for non-conserved quantities (liquid water, buoyancy, etc…)

What are your favorite invariants?

- I understand it…
- It makes it easy to compute buoyancy or density (e.g. M and D)
- It can be measured
- It can be conserved under specific diabatic transformation
- I can easily write its tendency equation under general (non-conservative) conditions



Buoyancy depends on location of cloud base



Efficiency of steam cycle depends on relative humidity

Unsaturated cycle

Buoyancy flux

- In the Boussinesq approximation, the generation of kinetic energy is given by the vertical integral of the buoyancy flux:

$$\frac{dKE}{dt} = \int \rho_0 \overline{w'B'} dz$$

- B is the buoyancy

$$B(S, r_T, z) = G \frac{\alpha(S, r_T, p_0(z)) - \alpha_0(z)}{\alpha_0(z)}$$

$$= G\rho_0(z)\left[\alpha(S, r_T, p_0(z)) - \alpha_0(z)\right]$$

- Linearize the buoyancy flux

$$\boxed{\rho_0 \overline{w'B'} \cong \left(\frac{\partial B}{\partial S}\right)_{r_T,p} \rho_0 \overline{w'S'} + \left(\frac{\partial B}{\partial r_T}\right)_{S,p} \rho_0 \overline{w'r_T{}'}}$$

with

$$B(S,r_T,z) = G\rho_0(z)\left[\alpha(S,r_T,p_0(z)) - \alpha_0(z)\right]$$

- The partial derivatives can be rewritten as

$$\left(\frac{\partial B}{\partial S}\right)_{r_T,p} = \rho_0 G\left(\frac{\partial \alpha}{\partial S}\right)_{r_T,p} = \frac{\rho_0 G}{1+r_{T0}}\left(\frac{\partial \alpha}{\partial S}\right)_{r_T,p}$$

$$\left(\frac{\partial B}{\partial r_T}\right)_{S,p} = \rho_0 G\left(\frac{\partial \alpha}{\partial r_T}\right)_{S,p} = \frac{\rho_0 G}{1+r_{T0}}\left(\frac{\partial \alpha_d}{\partial p}\right)_{r_T,S} - \frac{G}{1+r_{T0}}$$

$$\alpha_d = \frac{\alpha}{1+r_T}: \text{ specific volume per unit mass of DRY AIR}$$

- Maxwell relationships:

$$TdS = dH - \alpha_d dp - gdr$$

$$\alpha_d = \left(\frac{\partial H}{\partial p}\right)_{S,r_T}, \; T = \left(\frac{\partial H}{\partial S}\right)_{p,r_T} \; \text{and } g = \left(\frac{\partial H}{\partial r_T}\right)_{S,p}$$

$$\Rightarrow \quad \left(\frac{\partial \alpha_d}{\partial S}\right)_{p,r_T} = \left(\frac{\partial^2 H}{\partial S \partial p}\right)_{r_T} = \left(\frac{\partial T}{\partial p}\right)_{S,r_T}$$

$$\Rightarrow \quad \left(\frac{\partial \alpha_d}{\partial r_T}\right)_{p,S} = \left(\frac{\partial^2 H}{\partial r_T \partial p}\right)_{r_T} = \left(\frac{\partial g}{\partial p}\right)_{S,r_T}$$

- Maxwell relationships:

$$\left(\frac{\partial B}{\partial S}\right)_{r_T,p} = \frac{\rho_0 G}{1+r_{T0}}\left(\frac{\partial \alpha}{\partial S}\right)_{r_T,p} = \frac{\rho_0 G}{1+r_{T0}}\left(\frac{\partial T}{\partial p}\right)_{r_T,S}$$

$$= \frac{1}{1+r_{T0}}\Gamma_{ad}$$

$$\left(\frac{\partial B}{\partial r_T}\right)_{S,p} = \frac{\rho_0 G}{1+r_{T0}}\left(\frac{\partial \alpha_d}{\partial p}\right)_{r_T,S} - \frac{G}{1+r_{T0}}$$

$$= \frac{\rho_0 G}{1+r_{T0}}\left(\frac{\partial g}{\partial p}\right)_{r_T,S} - \frac{1}{1+r_{T0}}G$$

$$= -\frac{1}{1+r_{T0}}\left[\left(\frac{\partial g}{\partial z}\right)_{r_T,S} + G\right]$$

$$B' = \frac{1}{1+r_{T_0}}\Gamma_{ad}S' - \frac{1}{1+r_{T0}}\left[\frac{dg}{dz}+G\right]r_T{}'$$

$$\Rightarrow B' = \Gamma_{ad}S'' - \left[\frac{dg}{dz}+G\right]q_T{}'$$

S' : Total entropy perturbation per unit mass of dry air

$S'' = \dfrac{S'}{1+r_{T0}}$: Total entropy perturbation per unit mass of moist air

$q_T{}' = \dfrac{r_T{}'}{1+r_{T0}}$: Specific humidity perturbation (and not mixing ratio...)

$$\boxed{\overline{w'B'} = \Gamma_{ad}\overline{w'S''} - \left[\frac{dg}{dz}+G\right]\overline{w'q_T{}'}}$$

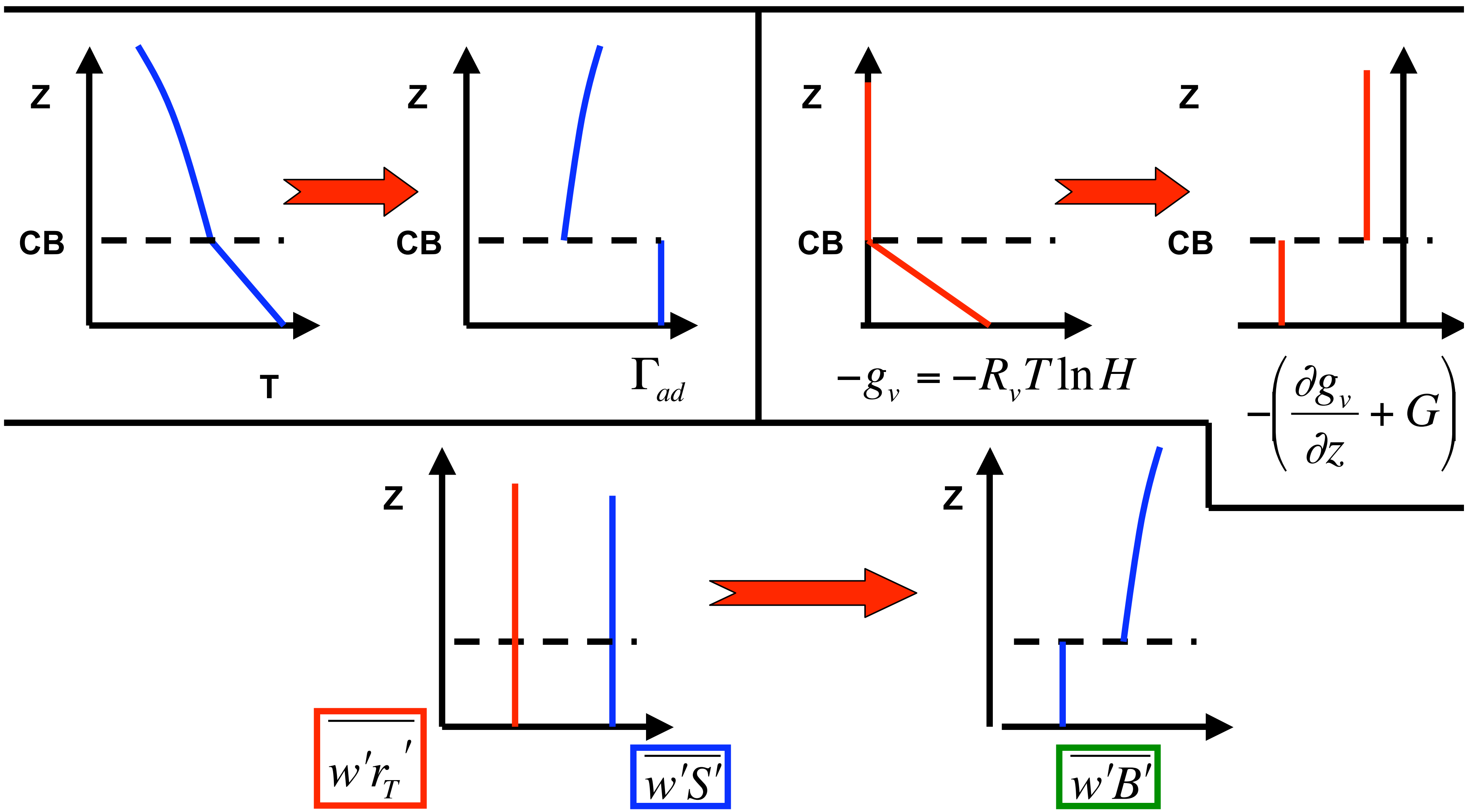
- After integration:

$$\frac{dKE}{dt}=\int_0^{\Delta z}\rho_0\overline{w'B'}dz$$

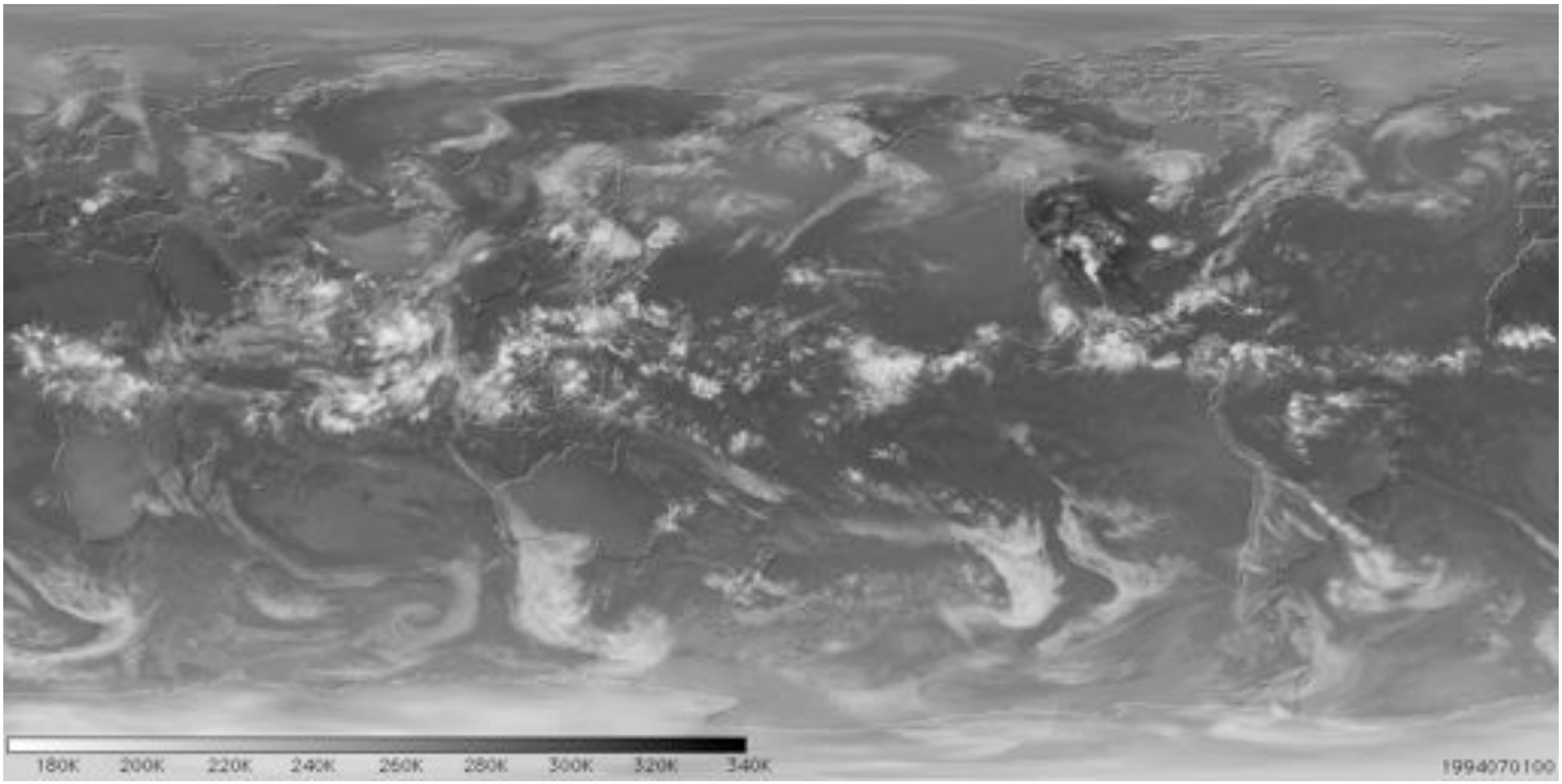
$$=\underbrace{(T_{in}-T_{out})\rho_0\overline{w'S''}}_{\text{Work done by a Carnot cycle}}+\underbrace{\left(g_{v,in}-g_{v,out}\right)\rho_0\overline{w'q_T{}'}}_{\text{“water vapor penalty”}}-\underbrace{G\Delta z\rho_0\overline{w'q_T{}'}}_{\text{geopotential energy gained by water}}$$

Work done by a Carnot cycle + “water vapor penalty” → Work done by a mixed Carnot-steam cycle

$$\rho_0 \overline{w'B'} \cong \Gamma_{ad} \rho_0 \overline{w'S'} - \left(\frac{\partial g_v}{\partial z} + G \right) \rho_0 \overline{w' r_T{}'}$$



Global considerations



- However, Columbus did not sail directly west from Spain. Rather, he went South to the Canaries islands.
- The prevailing winds in the Canaries blow from the East. This is what Columbus needed in order to sail West.

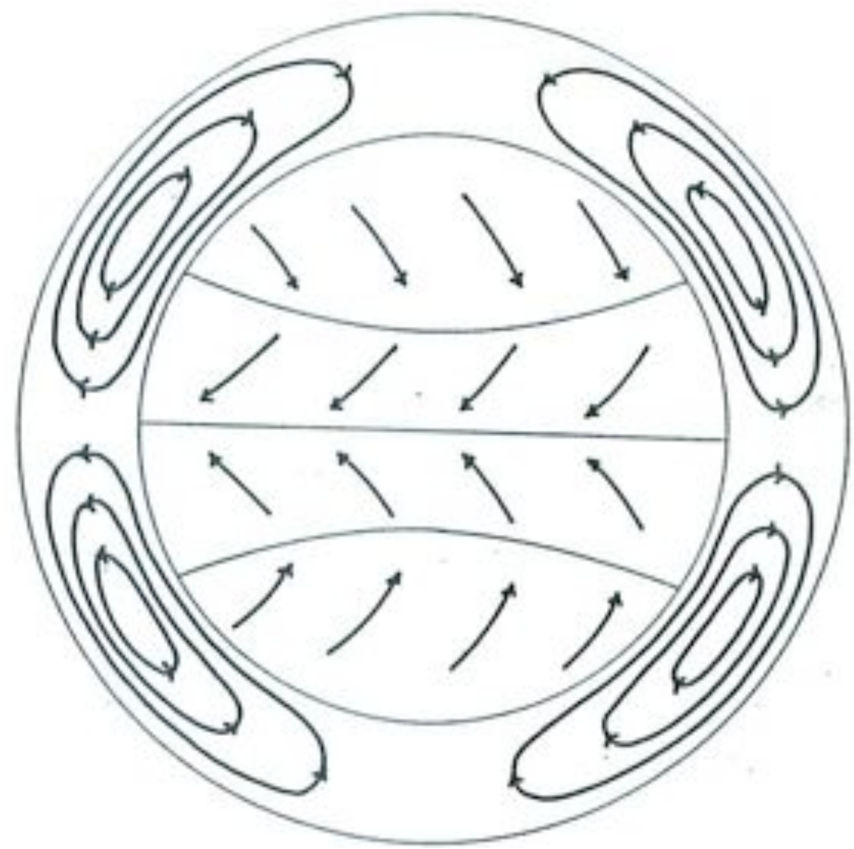
- These easterly winds are present through over all subtropical oceans.
- They are now known as the ‘Trade winds’ for the key role they played in the Transatlantic commerce.

I think that the causes of the General Trade-Winds have not been fully explained by those who have wrote on the subject…

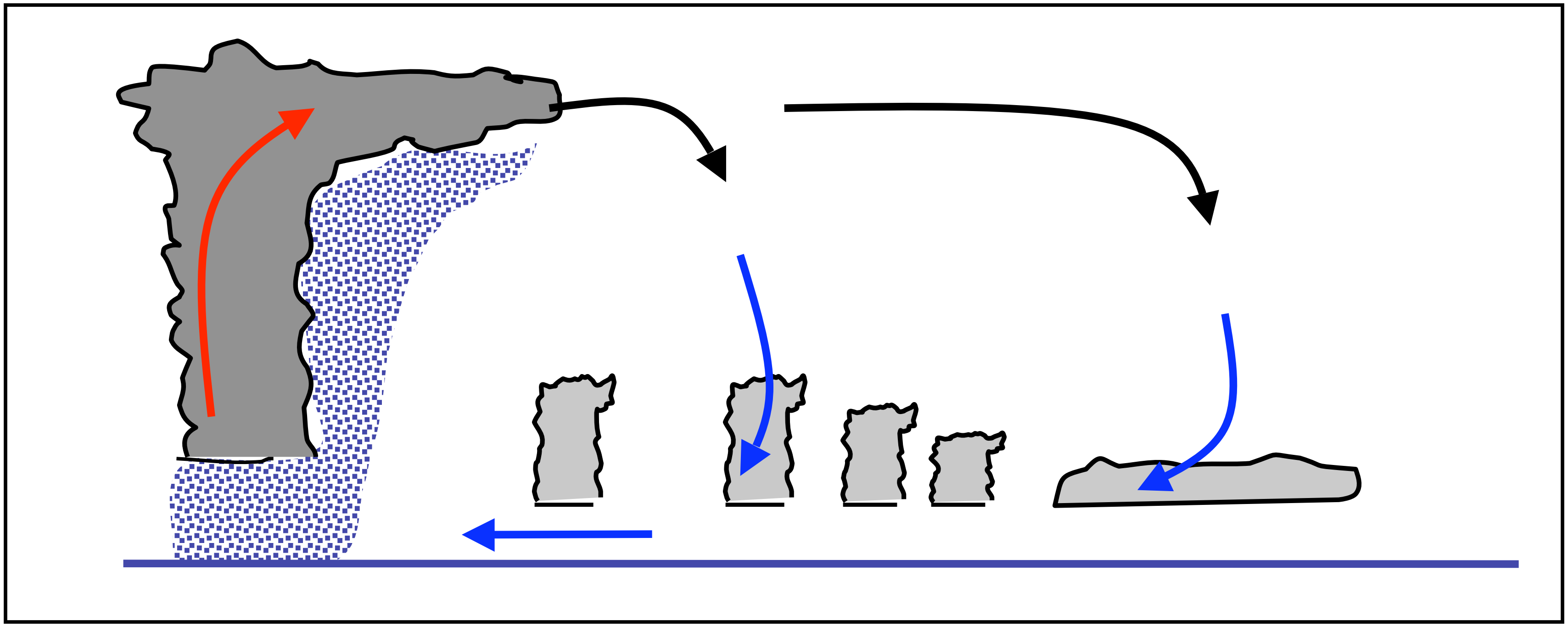
George Hadley (1735)

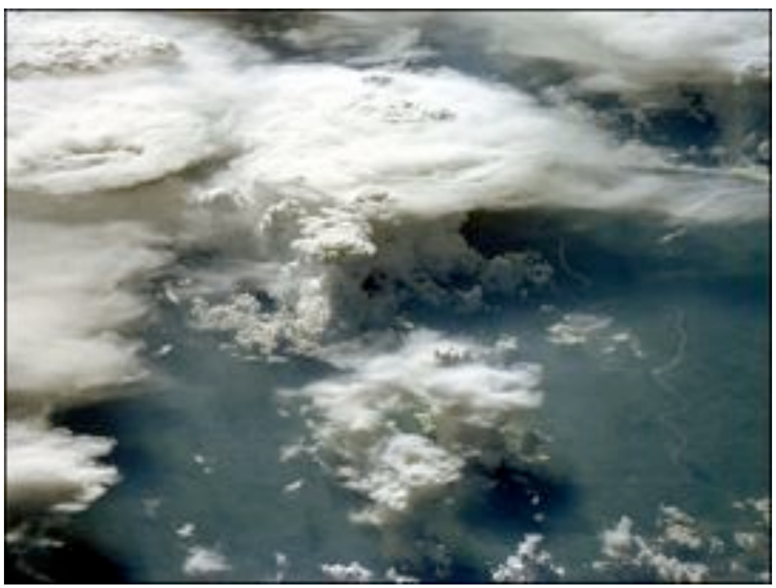
George Hadley (1735)

- Hadley explanation for the Trade winds:
  - There is a global circulation, with air rising at the Equator, and subsiding over the Poles
  - Conservation of (angular) momentum implies that air moving toward the equator acquires a easterly component.



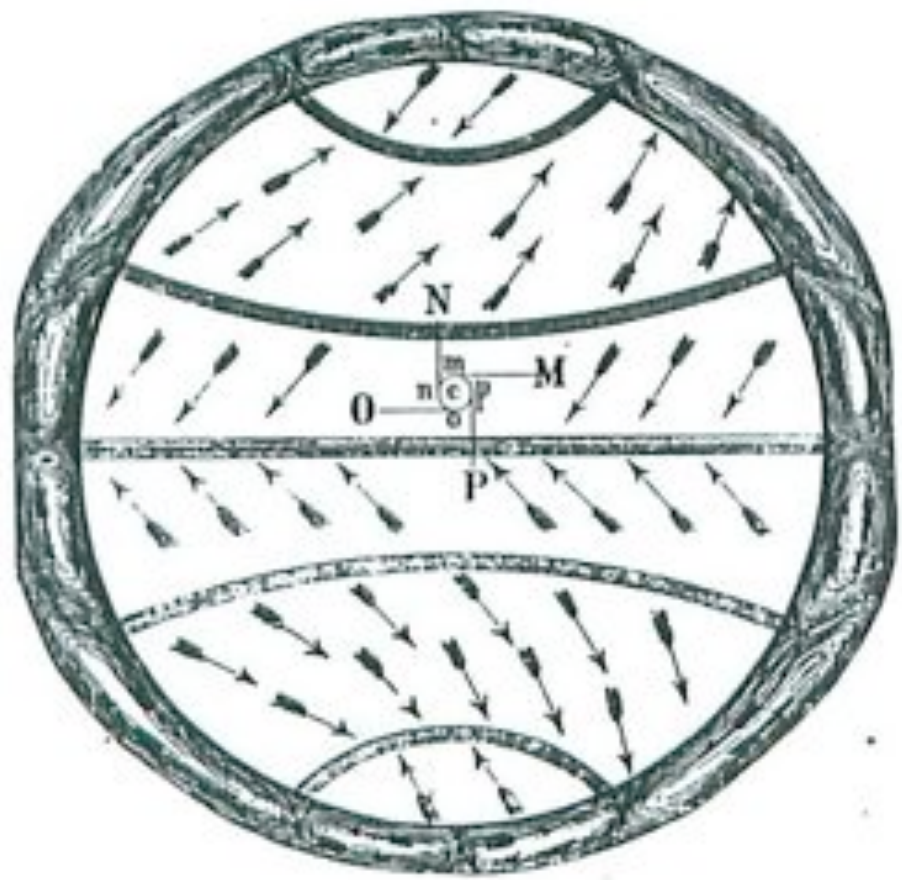
Clouds and the Hadley circulation





Ferrel (1836)

- Ferrel was the first to identify the role of rotation in atmospheric motions (the Coriolis effect).
- Also, using Maury's data, he identifies a reverse circulation associated with the westerly winds in the midlatitudes.



Bjerknes (1921)

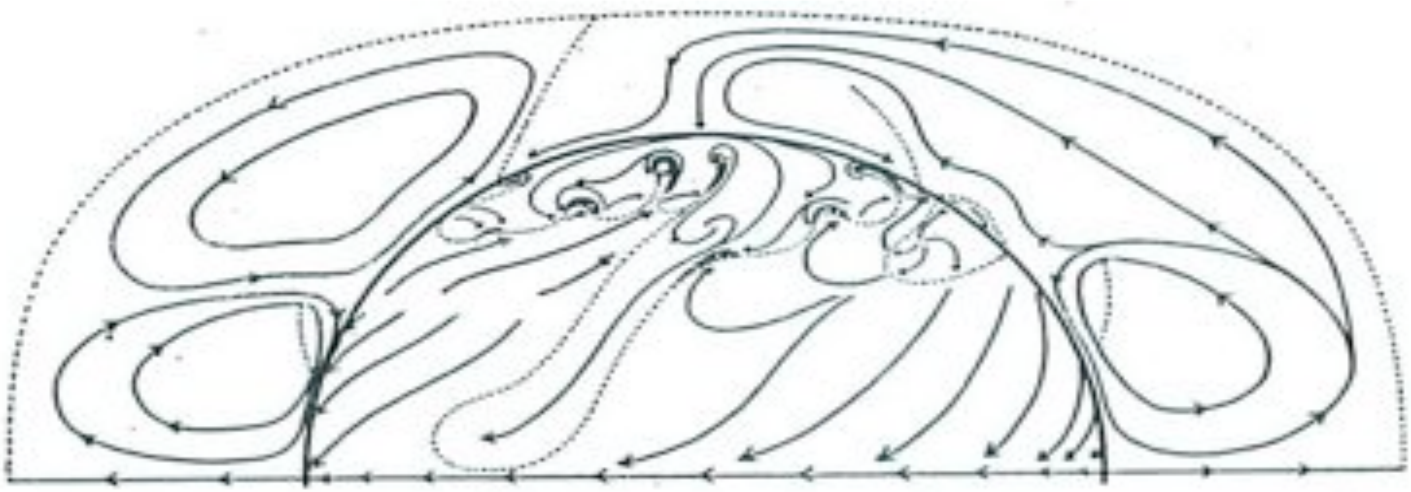
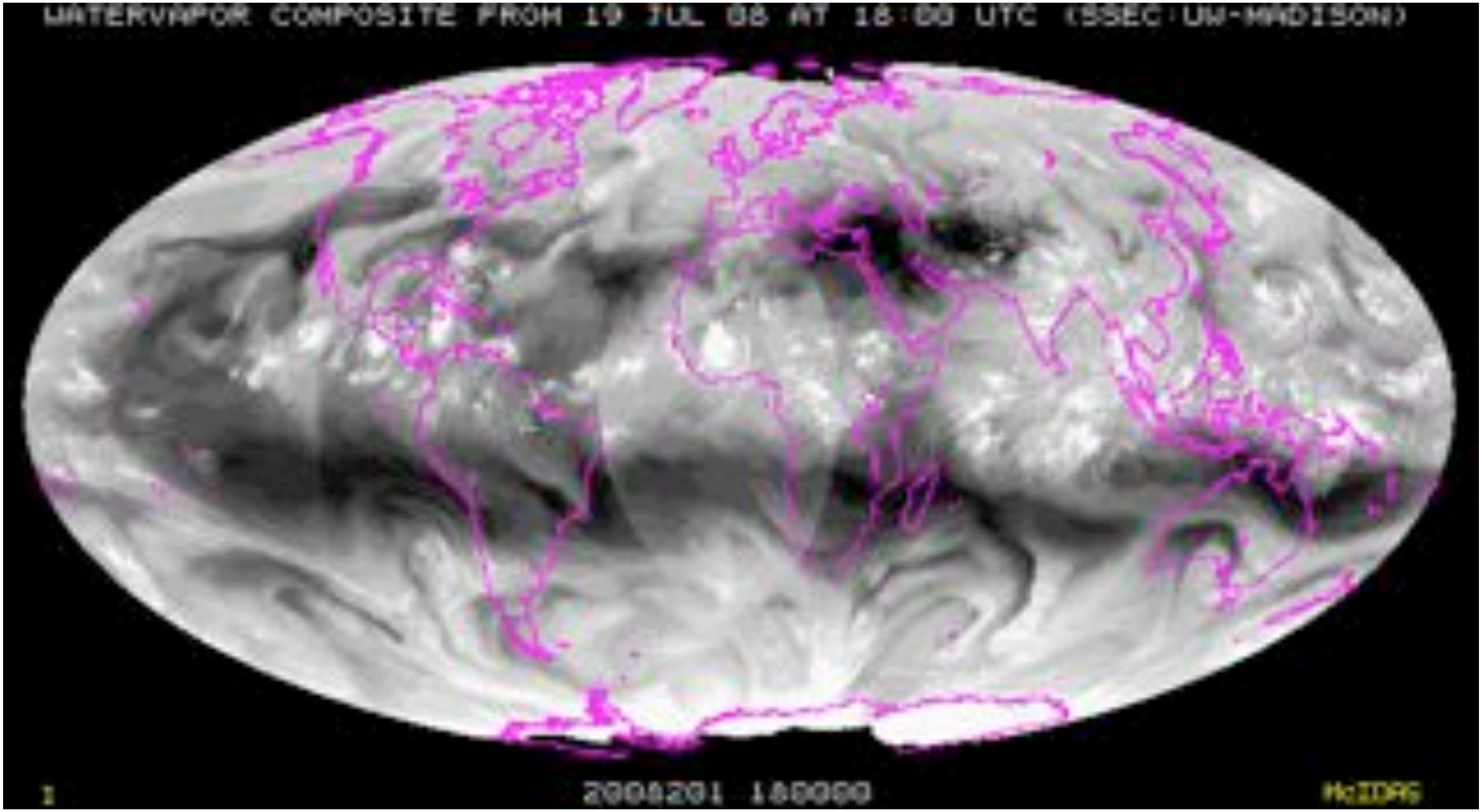


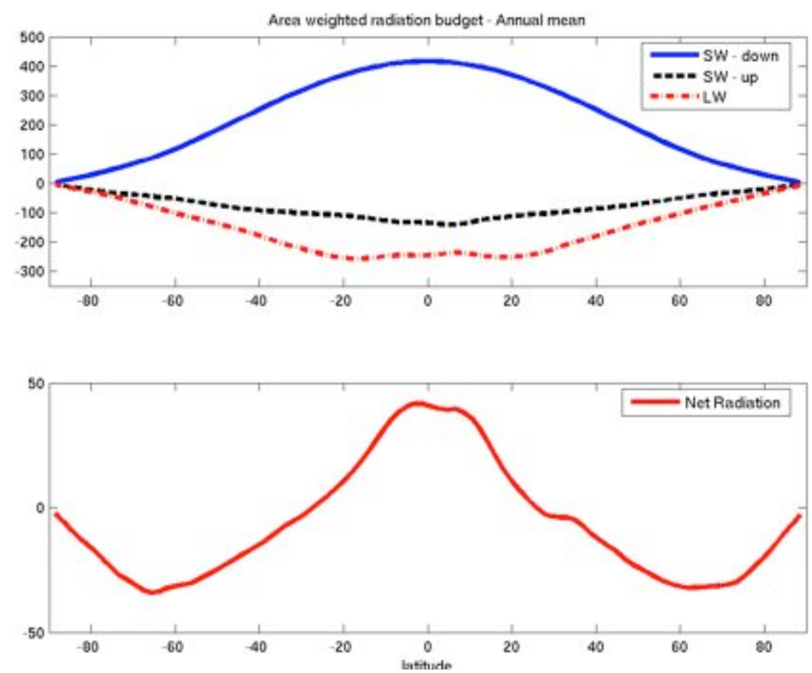
Figure 37. — A schematic representation of the general circulation of the atmosphere according to Bjerknes (1921)

- Midlatitudes are however dominated by storms (aka ‘synoptic scale eddies’).
- Understanding the interplay between the storm and global circulation is a key issue in modern meteorology.


WATERVAPOR COMPOSITE FROM 19 JUL 08 AT 18:00 UTC (SSEC UW-MADISON)
2008201 180000
McIDAS

The general circulation of the atmosphere

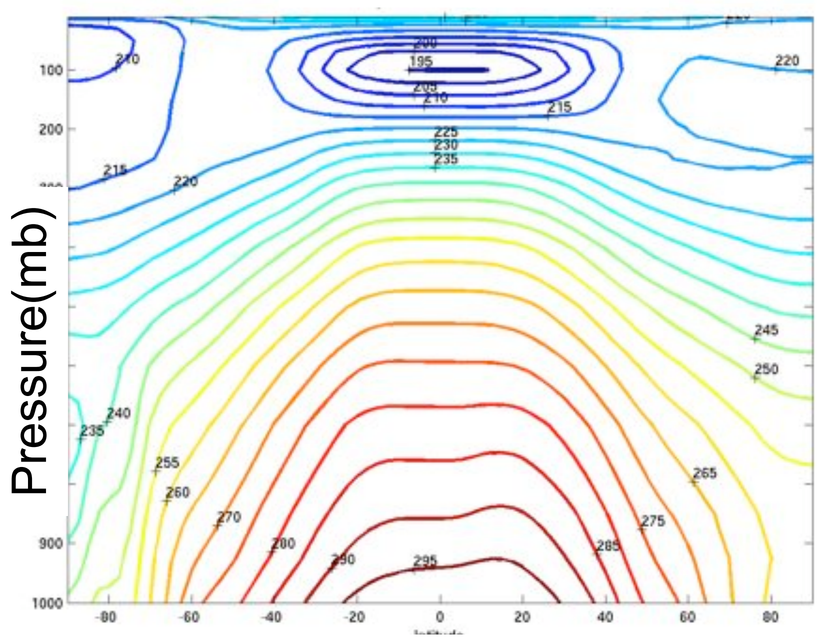
- The Earth's atmosphere receives most of its energy at the surface and in the Tropics.
- But it emits infra-red radiation rather uniformly.
- The circulation acts to transport energy from equator to Pole.
- Other important constraint related to angular momentum balance.



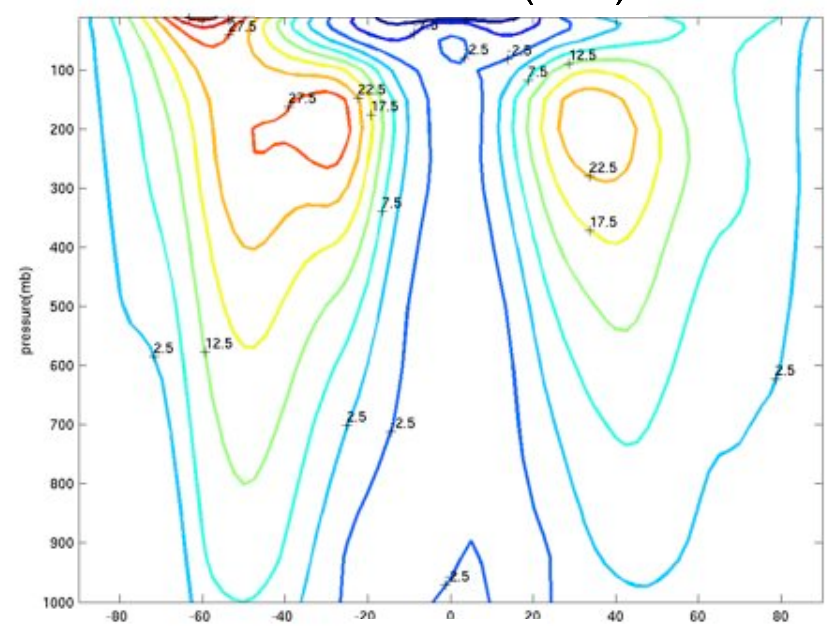
latitude

Eulerian averaging

Temperature (K)



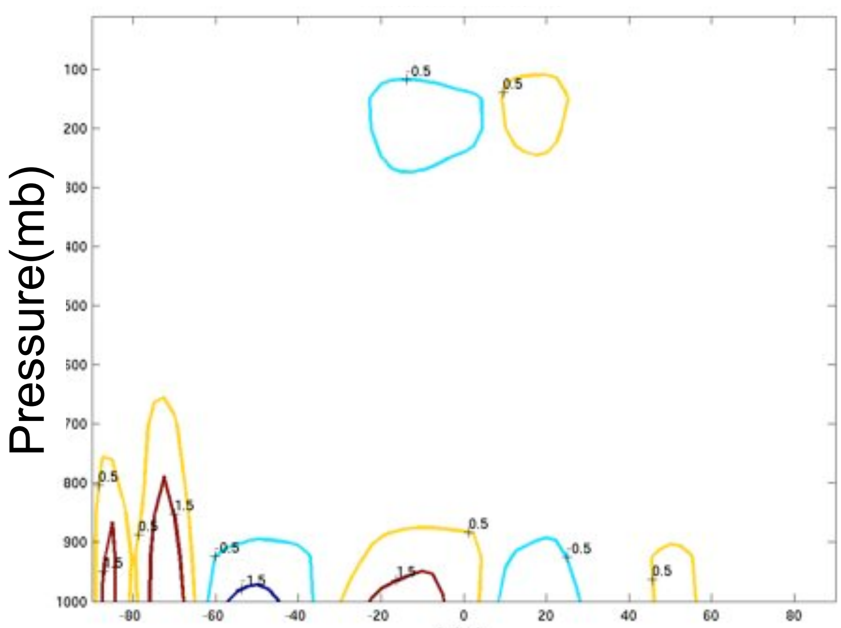
Zonal wind (m/s)

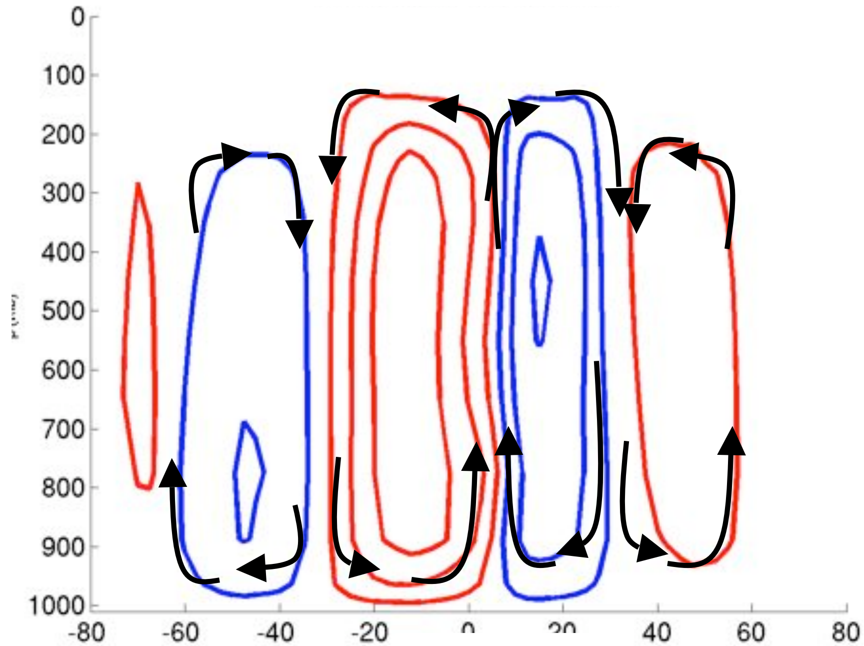


- Eulerian averaging: take the time and zonal average at fixed latitude and pressure (or height)

$$\overline{F}(\varphi,p) = \frac{1}{2\pi T}\int_0^T\int_0^{2\pi} F(\lambda,\varphi,p,t)\,d\lambda\,dt$$

Eulerian mean circulation



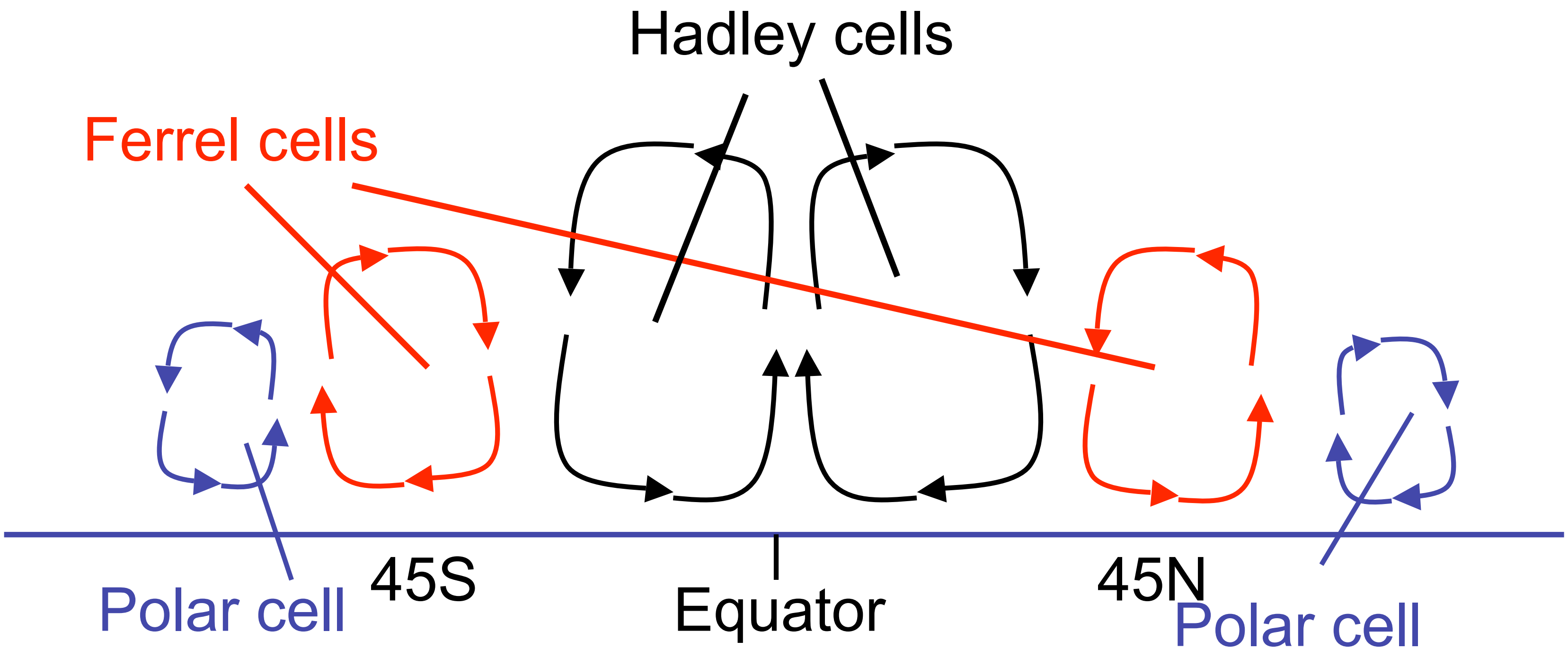


- Average velocity at constant pressure or height:

$$\bar{v}(\varphi,p) = \frac{1}{2\pi T}\int_0^T\int_0^{2\pi} v(\lambda,\varphi,p,t)\,d\lambda\,dt$$

- The circulation can be diagnosed by computing the stream function:

$$\Psi(\varphi,p) = \int_p^{p_{surf}} 2\pi\bar{v}a\cos\varphi\frac{dp}{g}$$



- Eulerian-mean circulation exhibits the ‘classic’ three-cell structure.
- But the Ferrel cell is a reverse circulation that transports energy toward the equator.

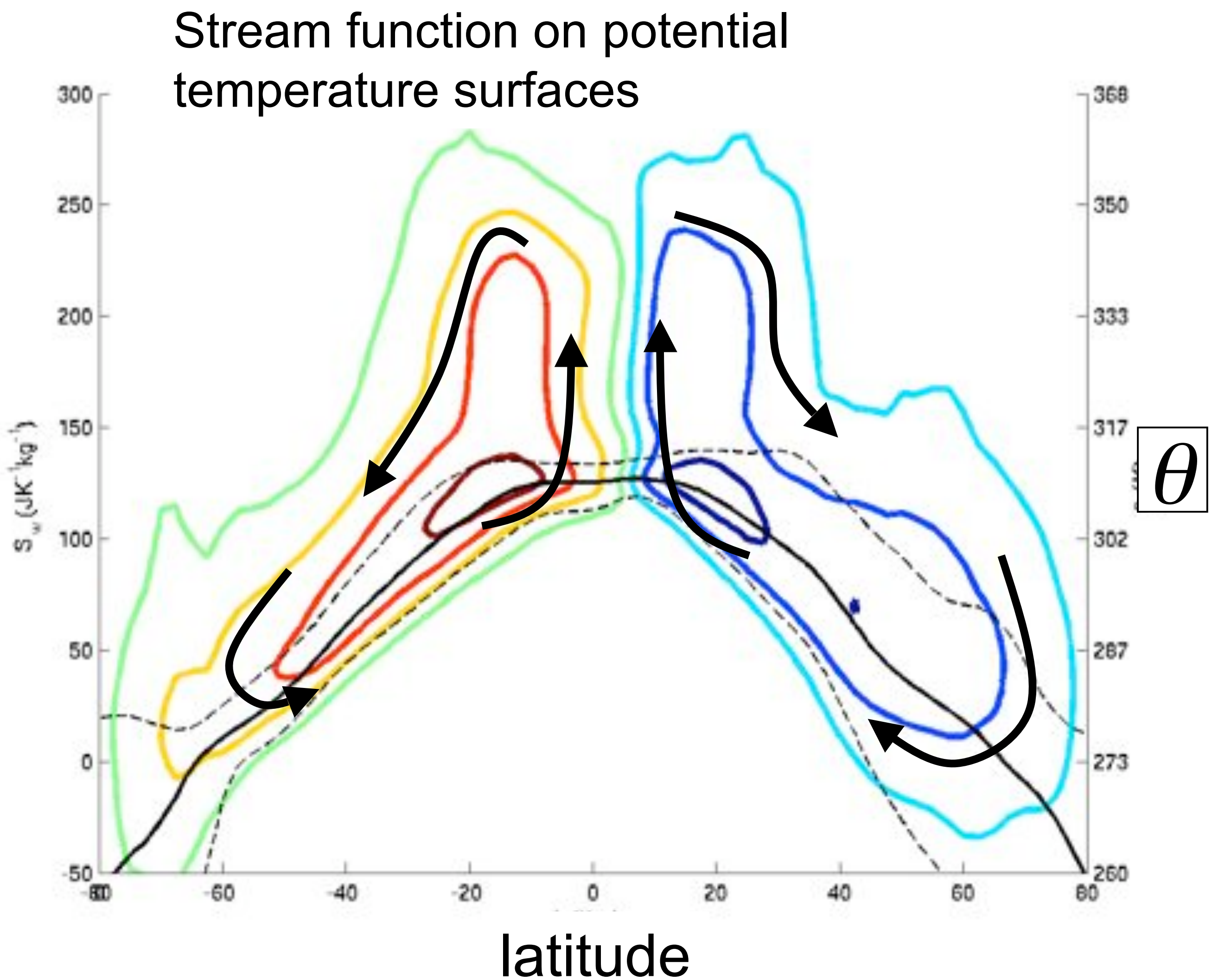
Circulation in isentropic coordinates (Dutton, Johnson, Held and Schneider)

- Rather than averaging in eulerian coordinates, one can average the circulation at constant value of the potential temperature θ:

$$\overline{F}^{\theta}(\varphi,\theta) = \frac{1}{2\pi T}\int_0^T \int_0^{2\pi} F(\lambda,\varphi,\theta,t)\,d\lambda\,dt$$

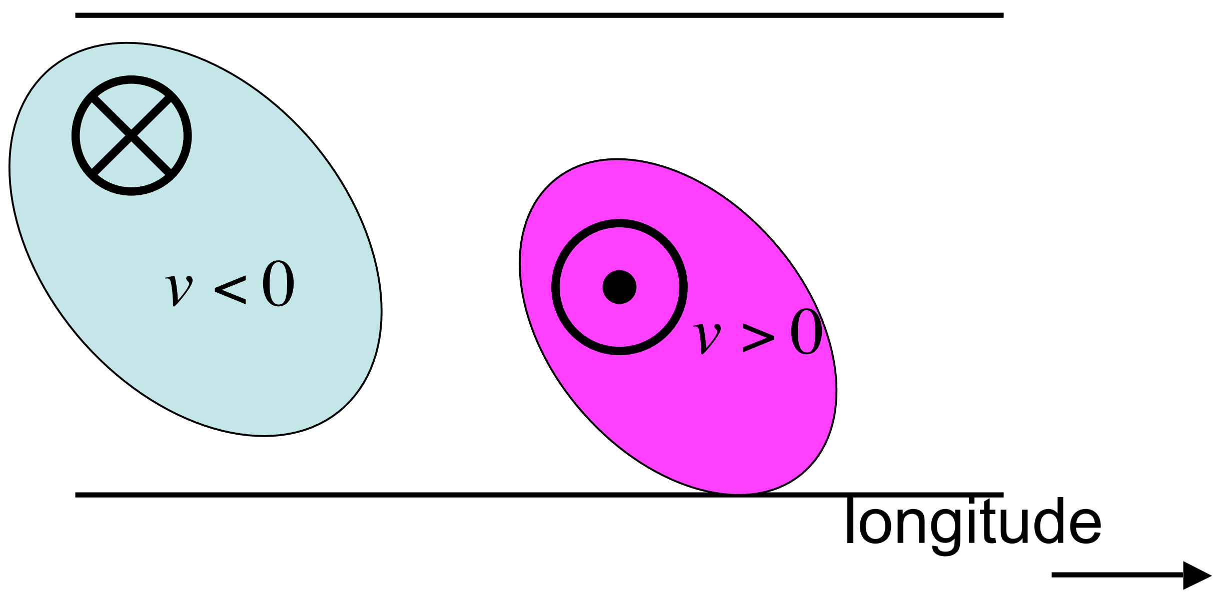
$$\Psi_{\theta}(\varphi,\theta) = \int_0^{\theta} 2\pi\,\overline{\rho_{\theta} v}^{\theta}\,a\cos\varphi\,d\theta$$

- Motivation: the potential temperature is related to entropy and is conserved for reversible adiabatic transformation in the absence of phase transition.



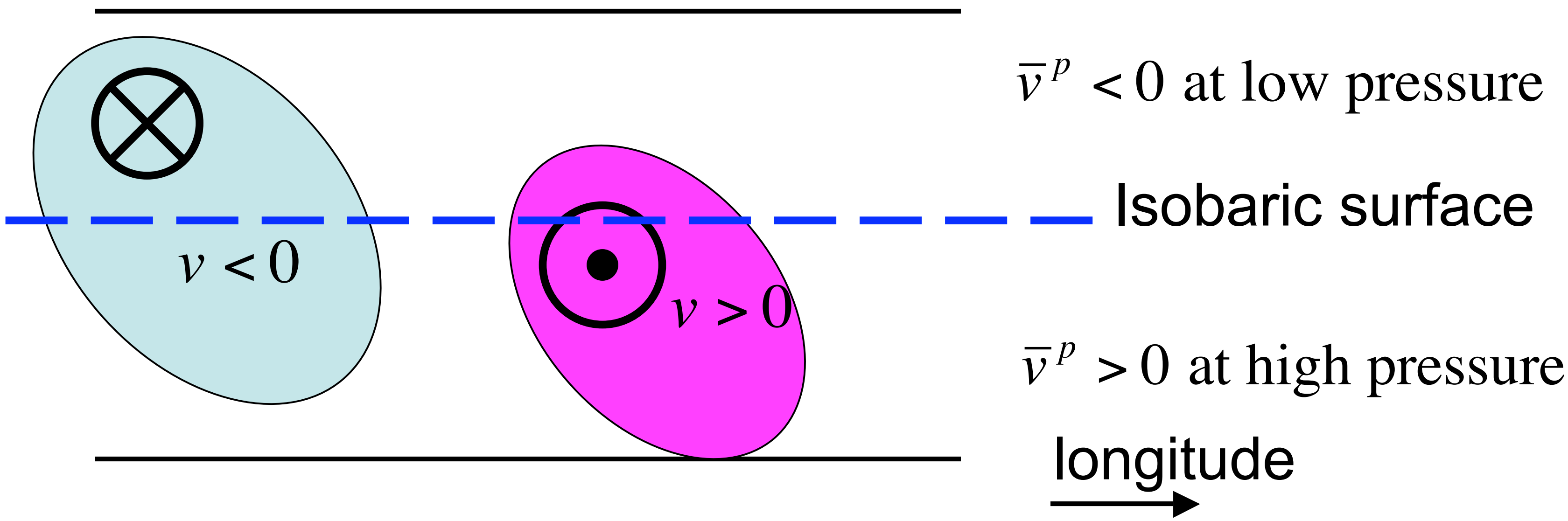
- The three cell structure disappears: there is a single Equator-to-Pole cell in each hemisphere.
- The circulation is direct (high entropy air flows poleward, low entropy air flow equatorward).

Why the circulation in eulerian and isentropic coordinates are in the opposite direction?



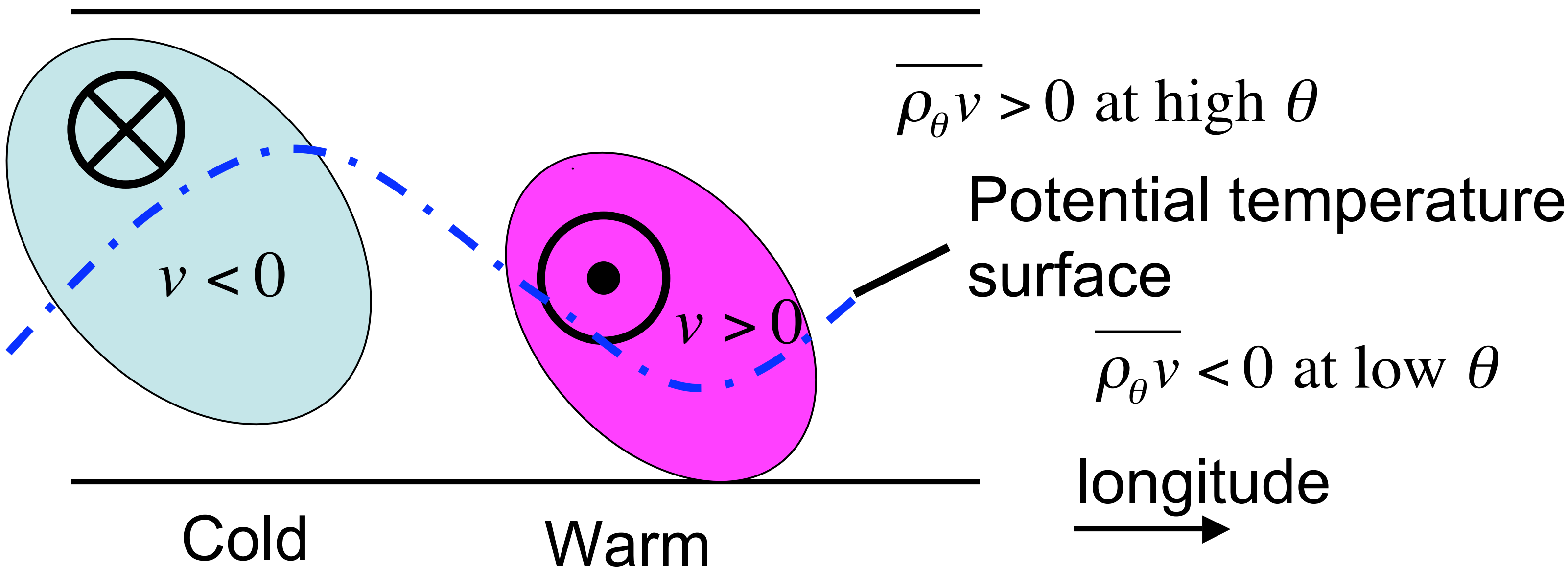
- In the midlatitudes, the flow is highly turbulent: the meridional velocity alternates between poleward and equatorward at all levels.

In the stormtracks: Eulerian-mean circulation



- In the midlatitudes, the flow is highly turbulent: the meridional velocity alternates between poleward and equatorward at all levels.
- This idealized eddies is associated with a poleward flow at high pressure/low level, and equatorward flow at high level

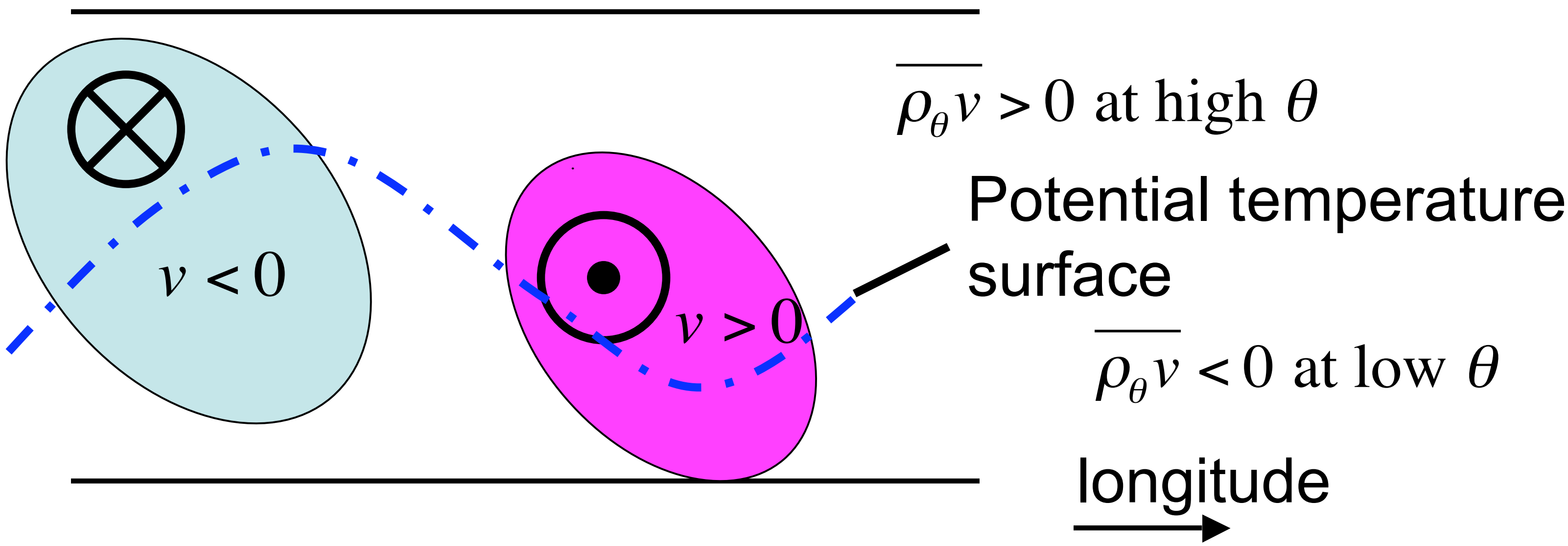
In the stormtracks:
Isentropic circulation



Thickness variations are such that the upper isentropic layer encompass larger fraction of the poleward flow.

Such pattern also corresponds to a net poleward energy mass transport.

Isentropic flow and eddy mass transport



- The mass flux on isentropic surfaces can be written as:

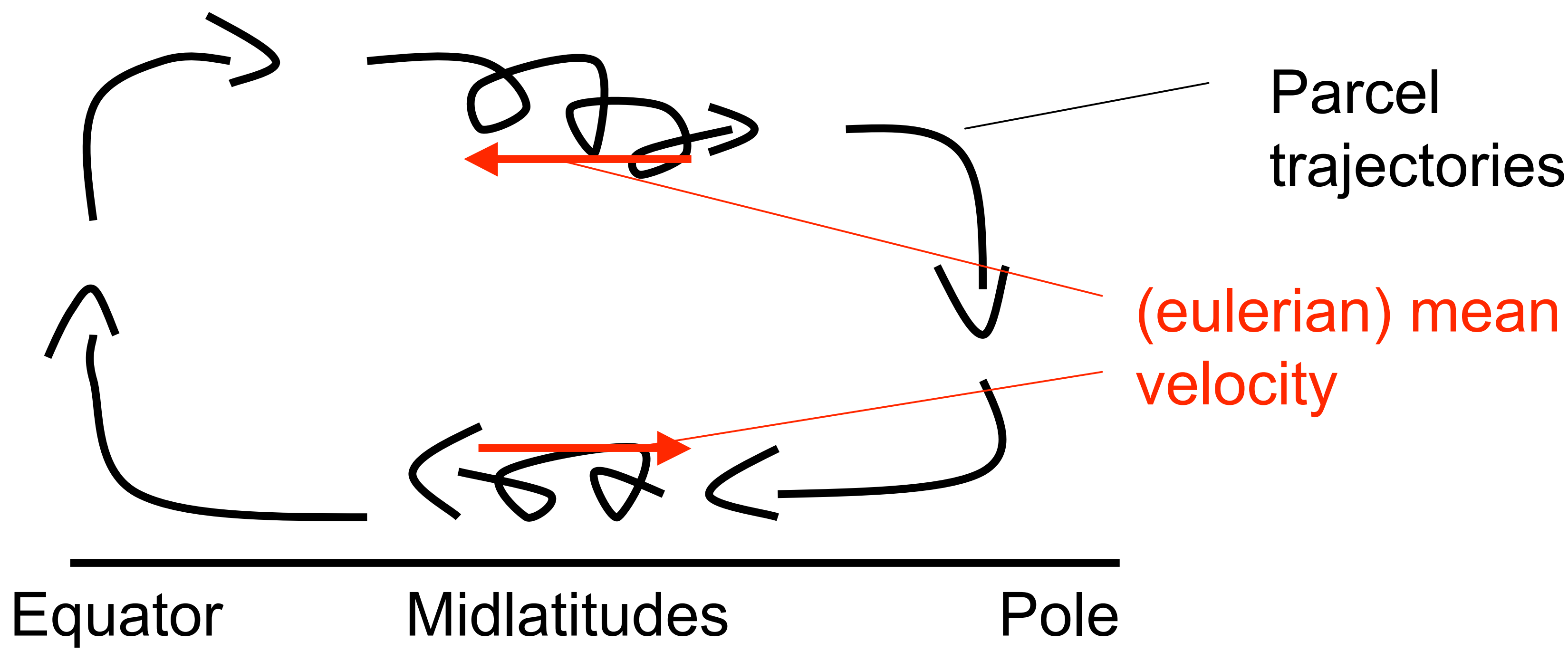
$$\overline{\rho_\theta v}^{\theta} = \overline{\rho_\theta}^{\theta}\,\overline{v}^{\theta} + \overline{\rho_\theta' v'}^{\theta}$$

Total mass flux

Transport by mean circulation

Eddy transport

- The mass transport by the eddies is in the opposite direction to the mean wind.

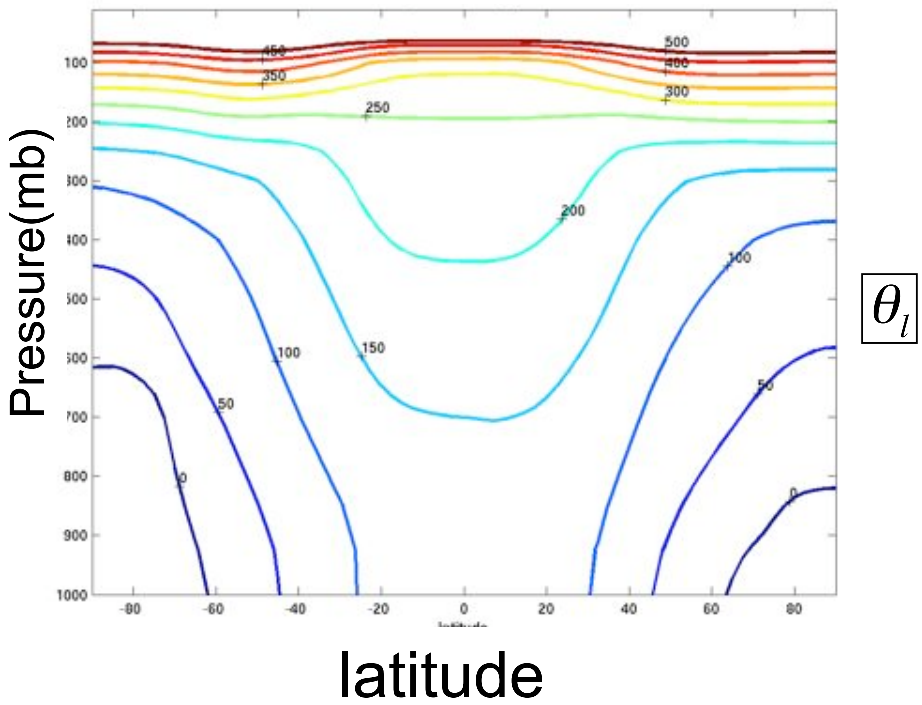


- The potential temperature is more or less conserved, and the the circulation on isentropes do a better job at capturing the mean lagrangian trajectories of air parcels.
- In the midlatitudes, the parcels move on average in the opposite direction to the (eulerian) mean velocity.

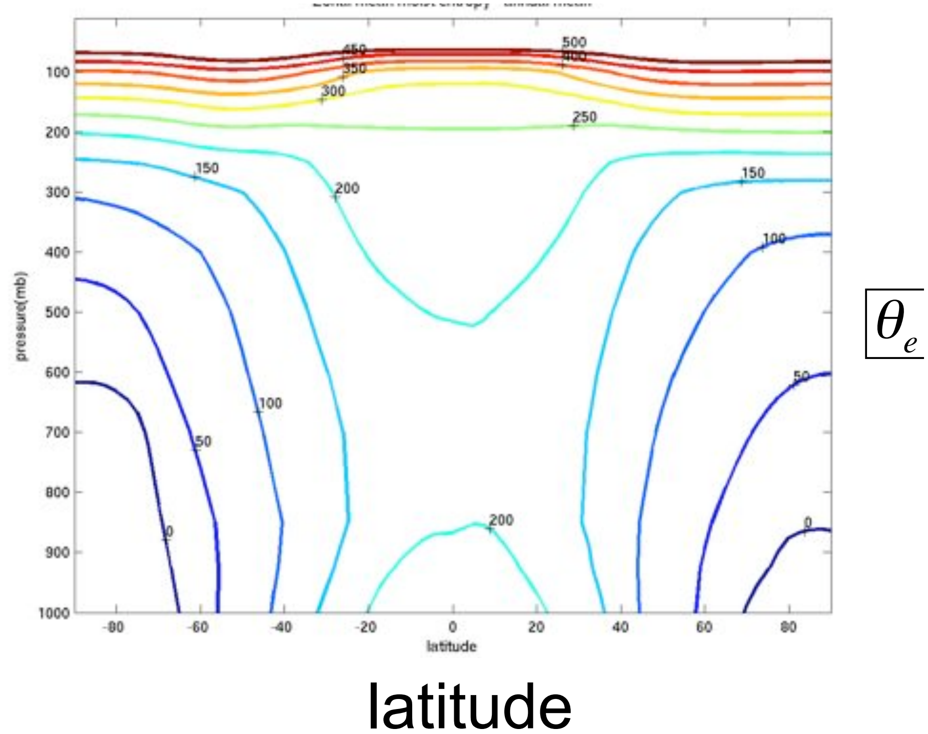
What about moisture?

- How to define an isentropic surface in a moist atmosphere?
- Previous studies have used the potential temperature θ as definition of entropy.
- The equivalent potential temperature θ_e is conserved for reversible adiabatic transformation, even when phase transition take place.
- Why not use θ_e then?
- Does it matter?

'Dry isentropes': $\theta_l = cst$

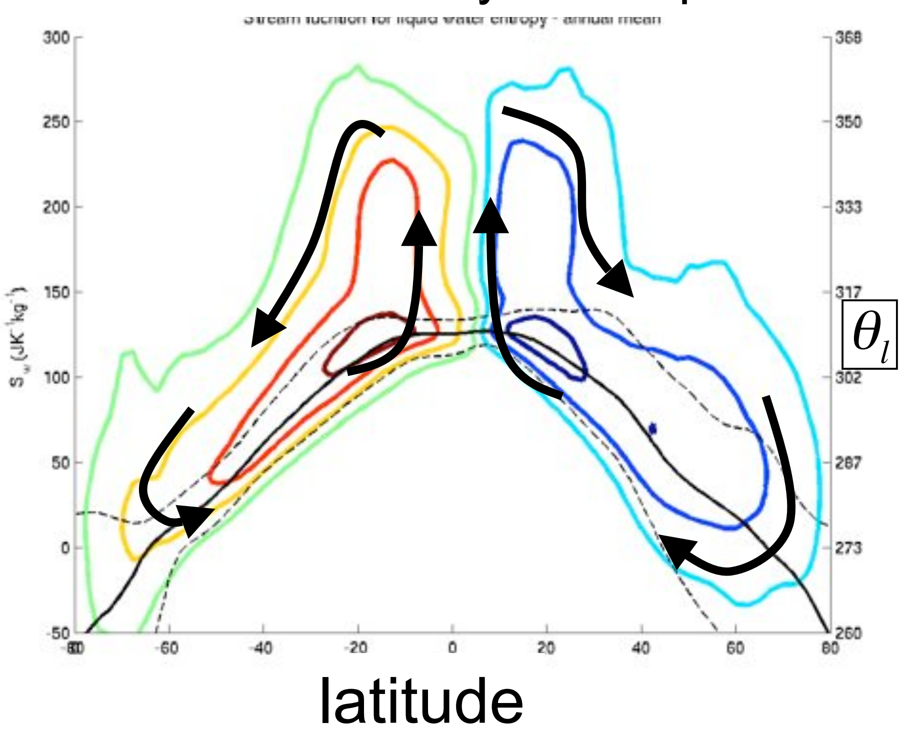


'Moist isentropes': $\theta_e = cst$

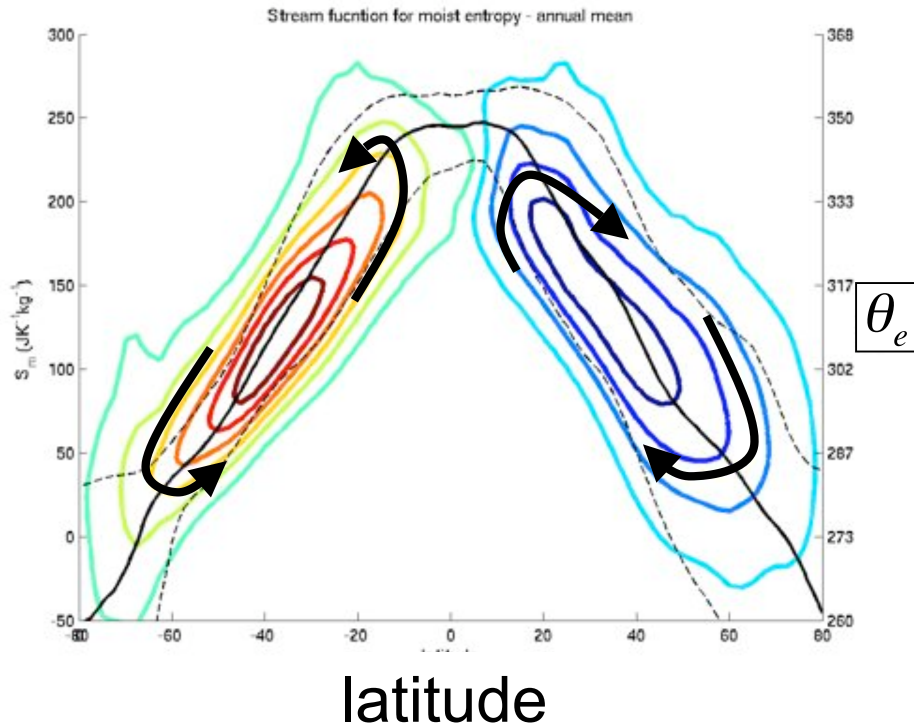


- Different definitions imply different isentropic surfaces.
- θ_e includes a contribution from the latent heat content, and has often a minimum in the middle of the atmosphere.

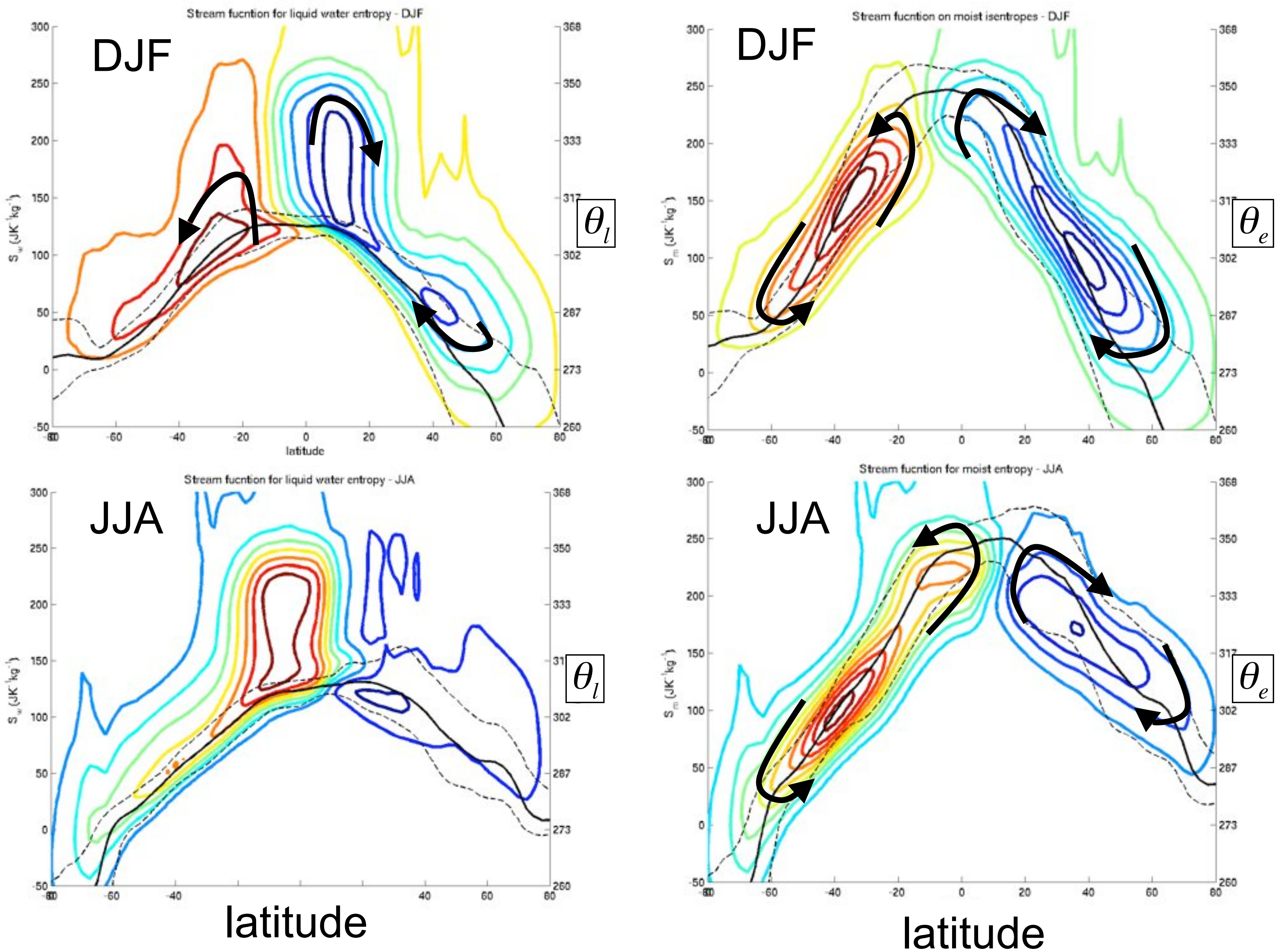
Stream function on dry isentropes



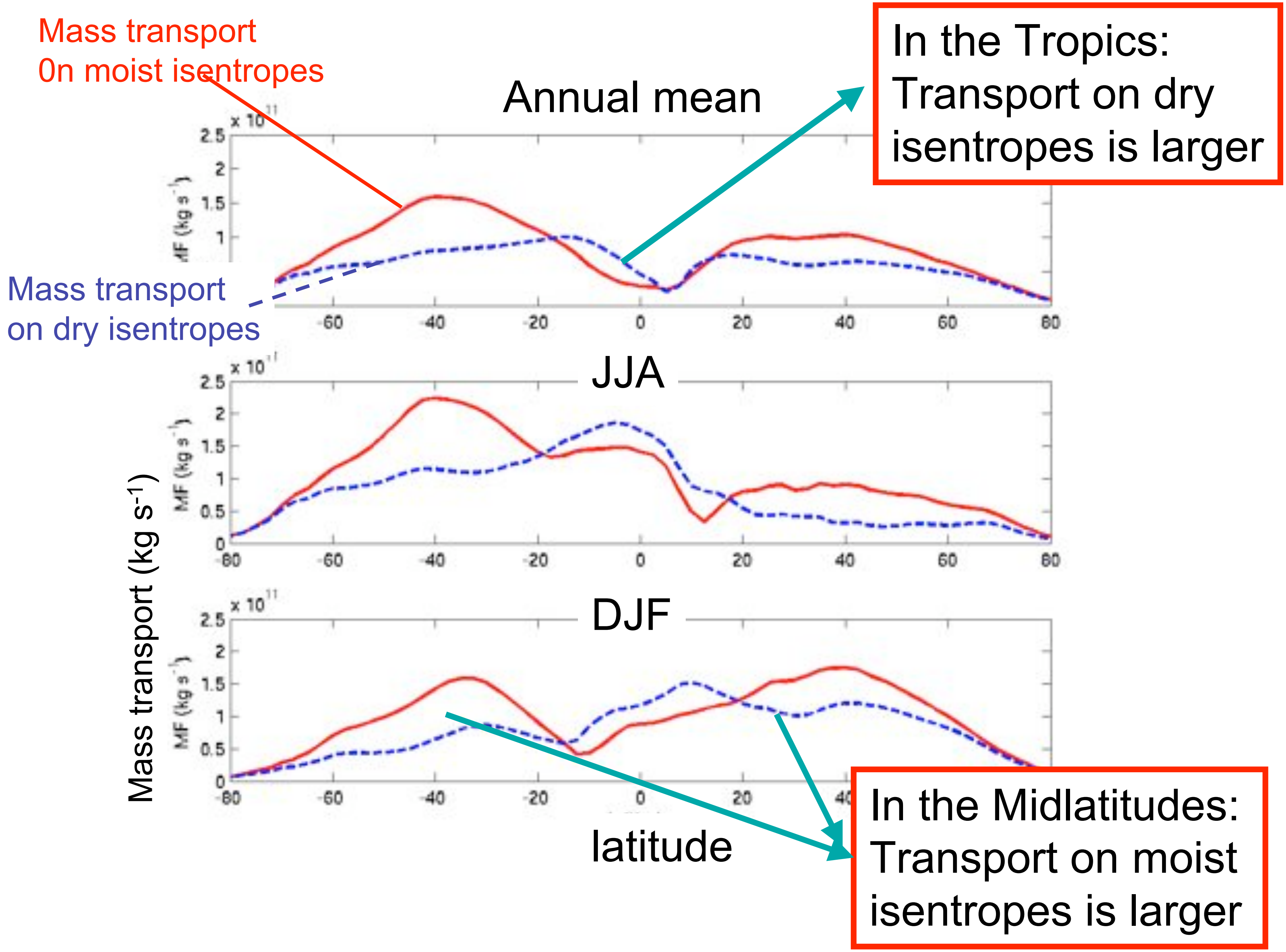
Stream function on moist isentropes



- Same single cell structure…
- But amplitude of the circulation differs!



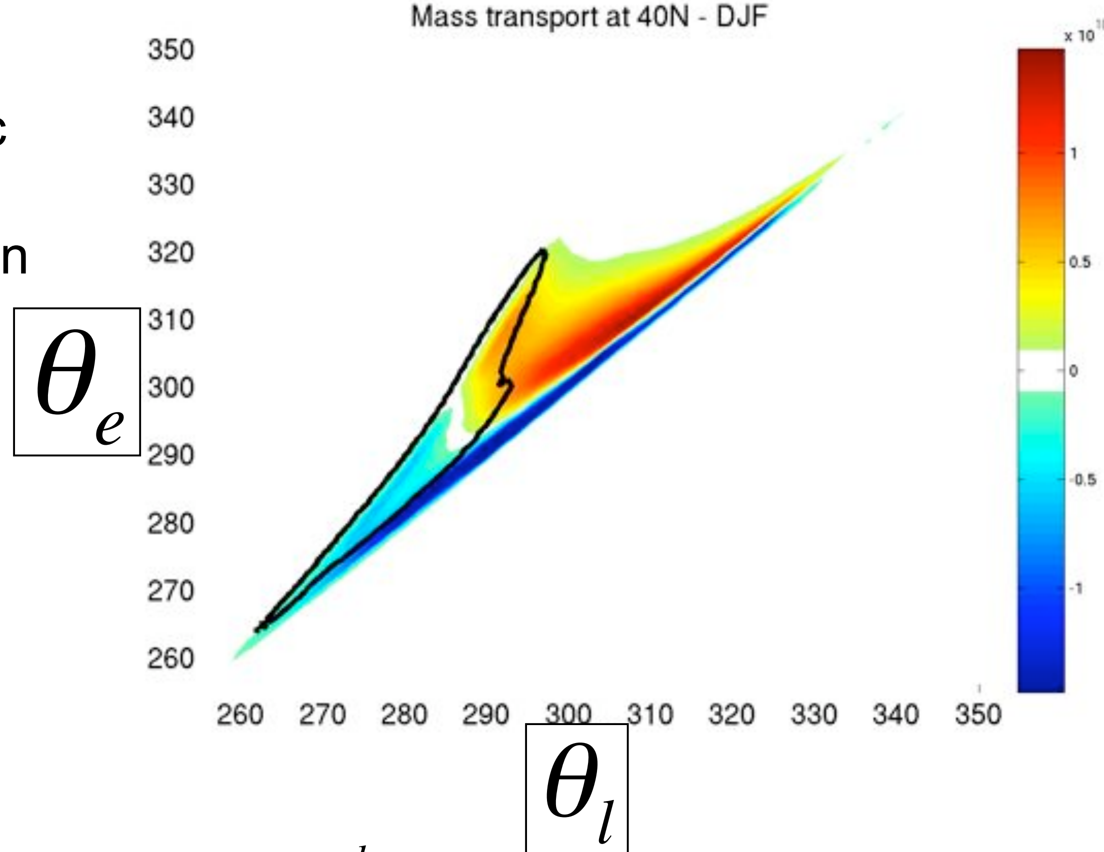
Stream fucntion for liquid water entropy - DJF
DJF
θ_l
latitude
Stream fucntion on moist isentropes - DJF
DJF
θ_e
Stream fucntion for liquid water entropy - JJA
JJA
θ_l
Stream fucntion for moist entropy - JJA
JJA
θ_e
latitude
latitude



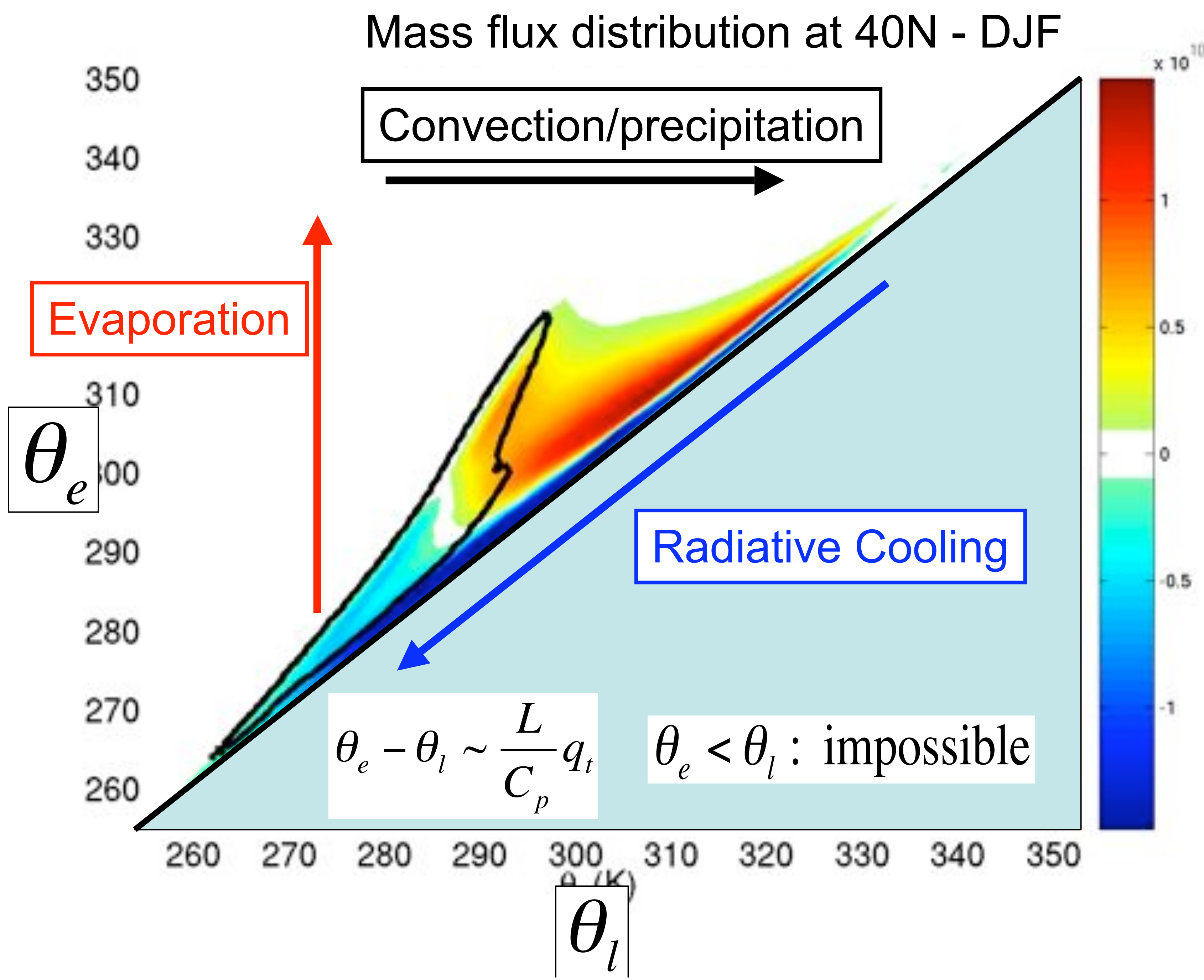
Mass transport
0n moist isentropes
Annual mean
In the Tropics:
Transport on dry
isentropes is larger
Mass transport
on dry isentropes
JJA
DJF
Mass transport (kg s-1)
latitude
In the Midlatitudes:
Transport on moist
isentropes is larger

Why the difference in mass transport?

- Both θ_e and θ_l are conserved along adiabatic trajectories. Rather than isentropic surfaces, we can think of having a set of 'isentropic filaments' -i.e. lines of constant value for both θ_e and θ_l.
- The poleward mass transport along such isentropic filaments is defined as:

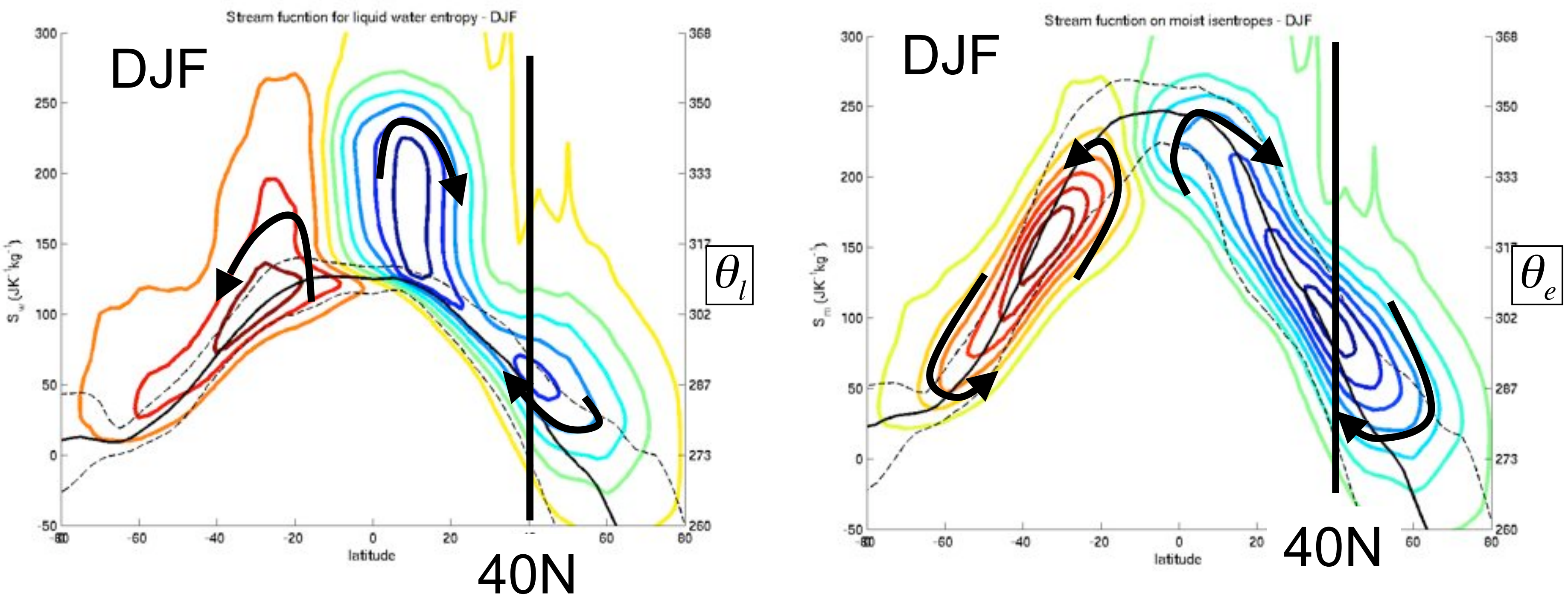


$$M(\theta_{l0},\theta_{e0},\varphi) = \frac{1}{2\pi T}\int\int\int \rho v \delta(\theta_l - \theta_{l0})\delta(\theta_e - \theta_{e0}) d\lambda \frac{dp}{g} dt$$



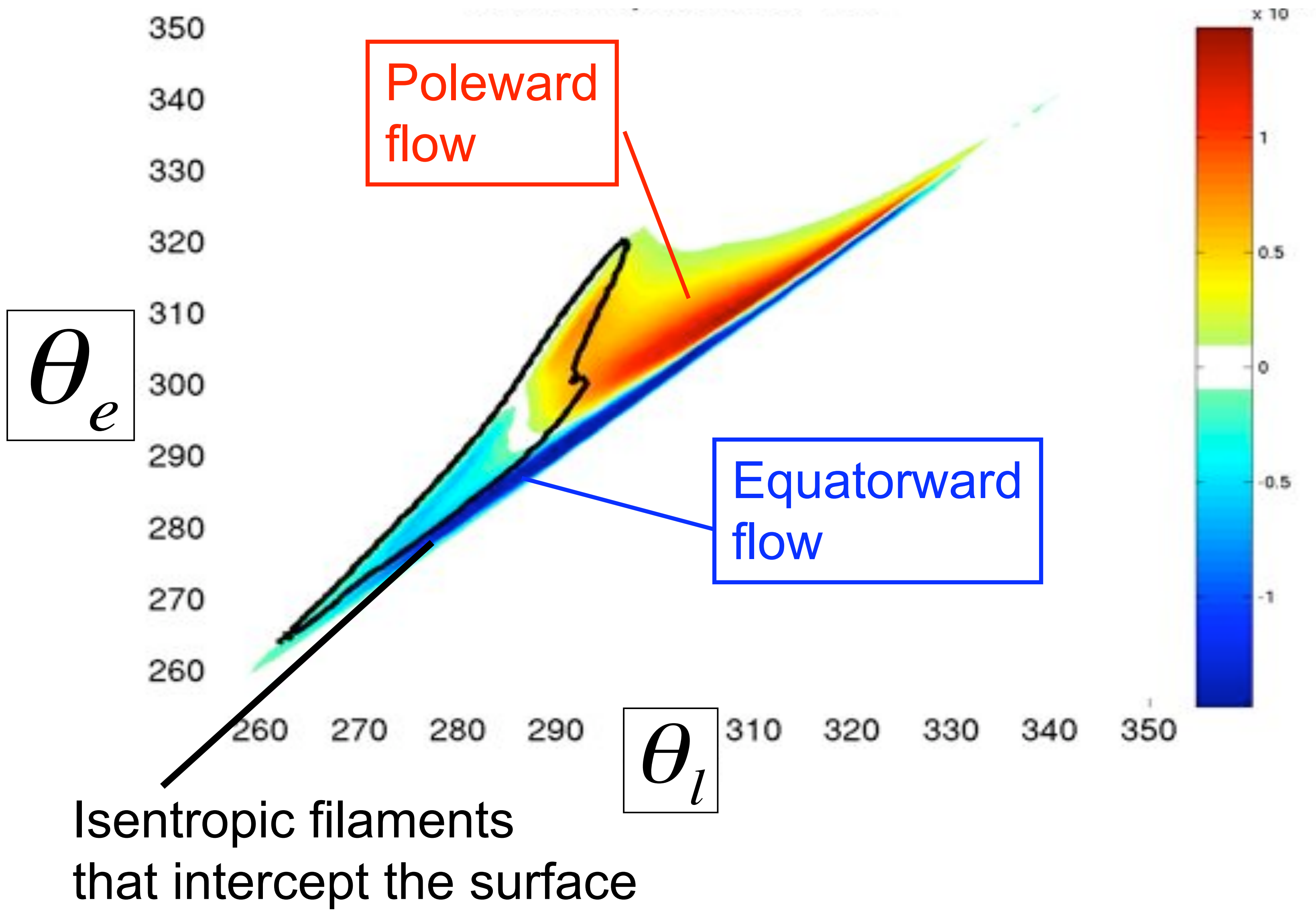
Mass flux distribution at 40N - DJF
Convection/precipitation
Evaporation
Radiative Cooling
$\theta_e - \theta_l \sim \frac{L}{C_p} q_t$
$\theta_e < \theta_l$: impossible
θ_e
θ_l
350
340
330
310
290
280
270
260
300
320
x 10^10
1
0.5
0
-0.5
-1

In the Midlatitudes:



- Circulation on moist isentropes is larger than that on dry isentropes.

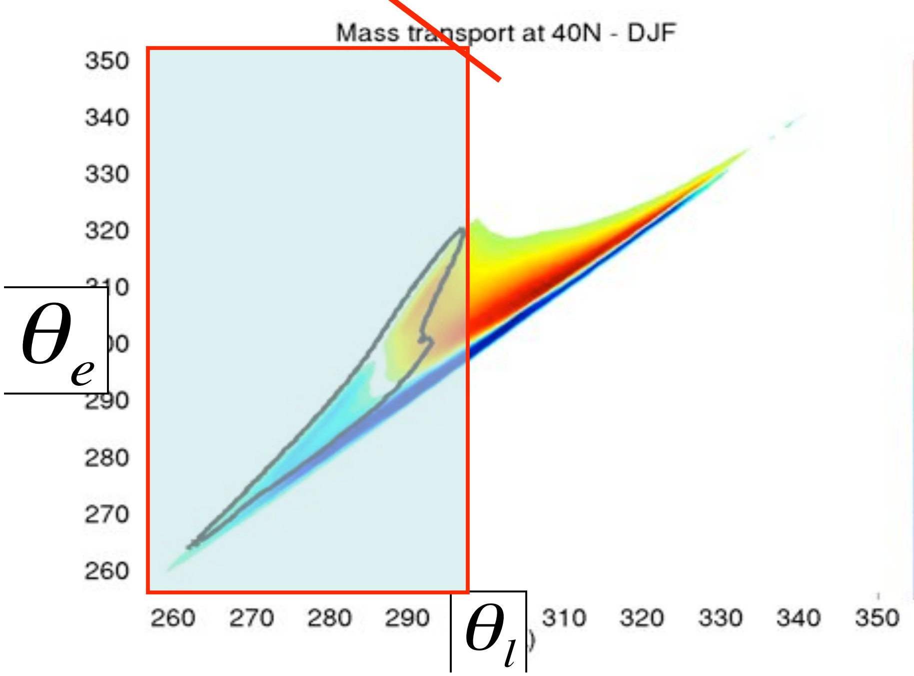
Mass flux distribution at 40N - DJF



Mass flux and stream function at 40N

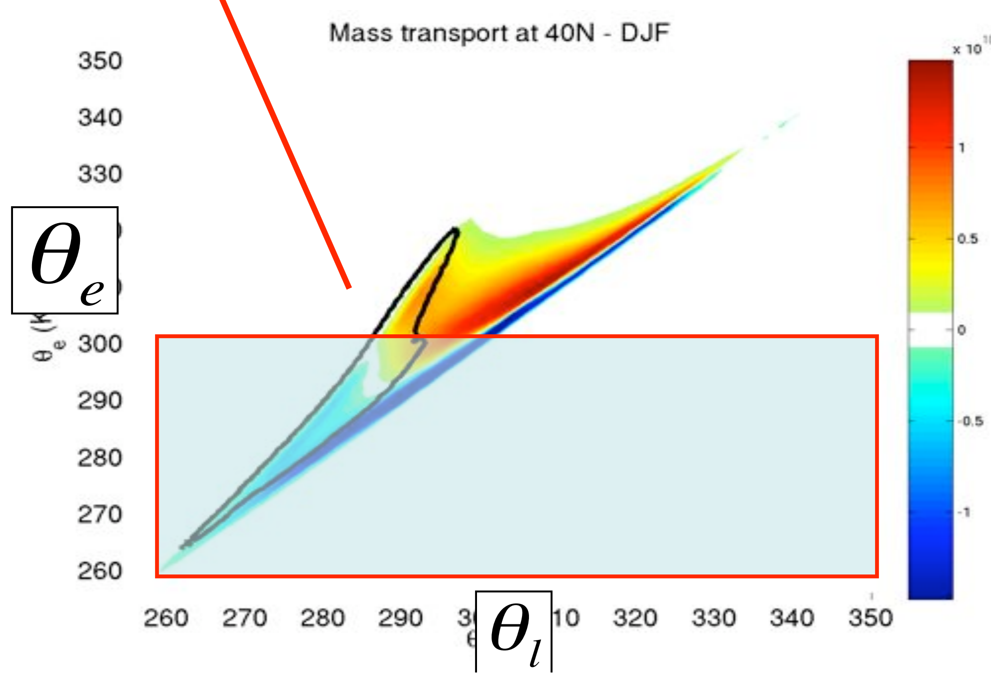
Stream function on dry isentropes:

$$\Psi_{\theta_l}(\Theta) = \int_0^{\Theta} \int_0^{\infty} M(\theta_l,\theta_e)\,d\theta_e\,d\theta_l$$

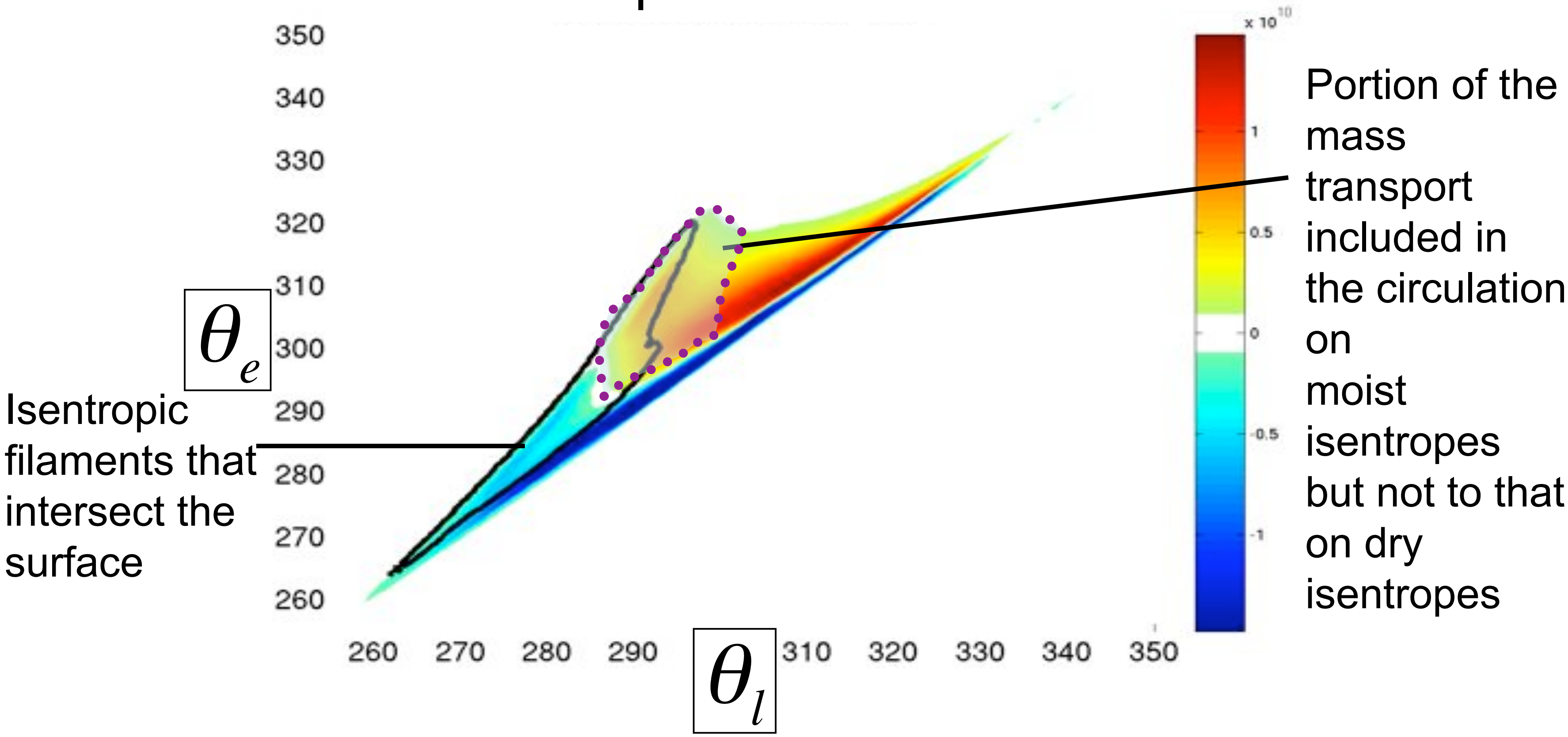


Stream function on moist isentropes:

$$\Psi_{\theta_e}(\Theta) = \int_0^{\Theta} \int_0^{\infty} M(\theta_l,\theta_e)\,d\theta_l\,d\theta_e$$

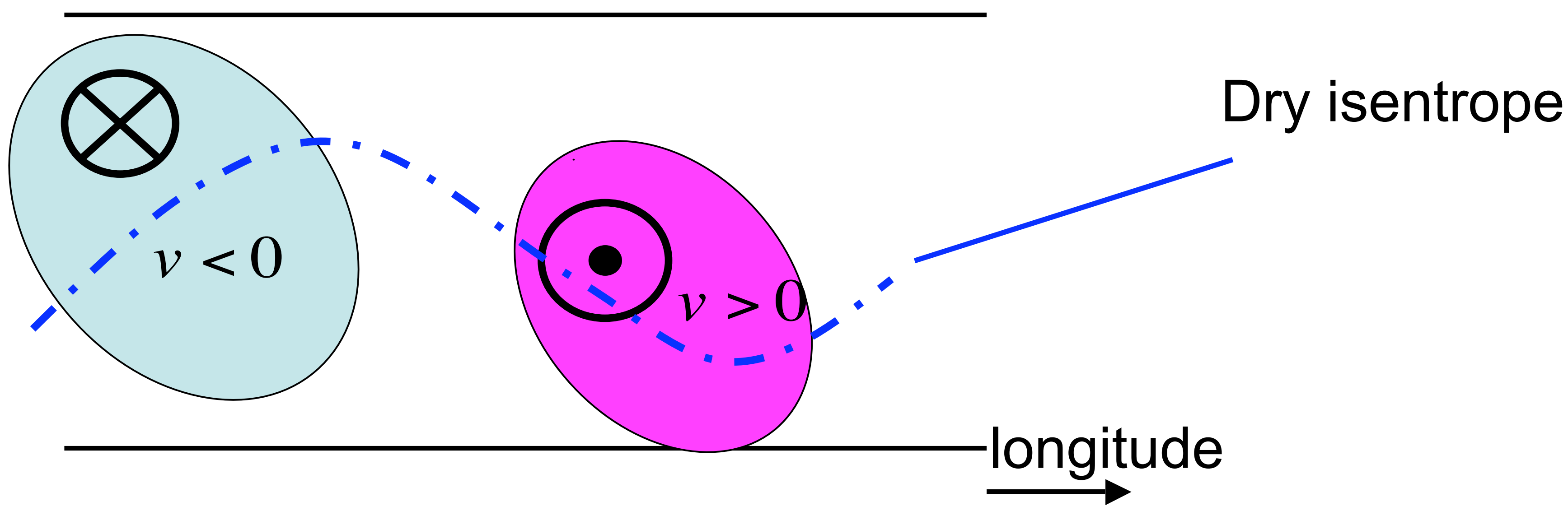


Mass transport at 40N - DJF

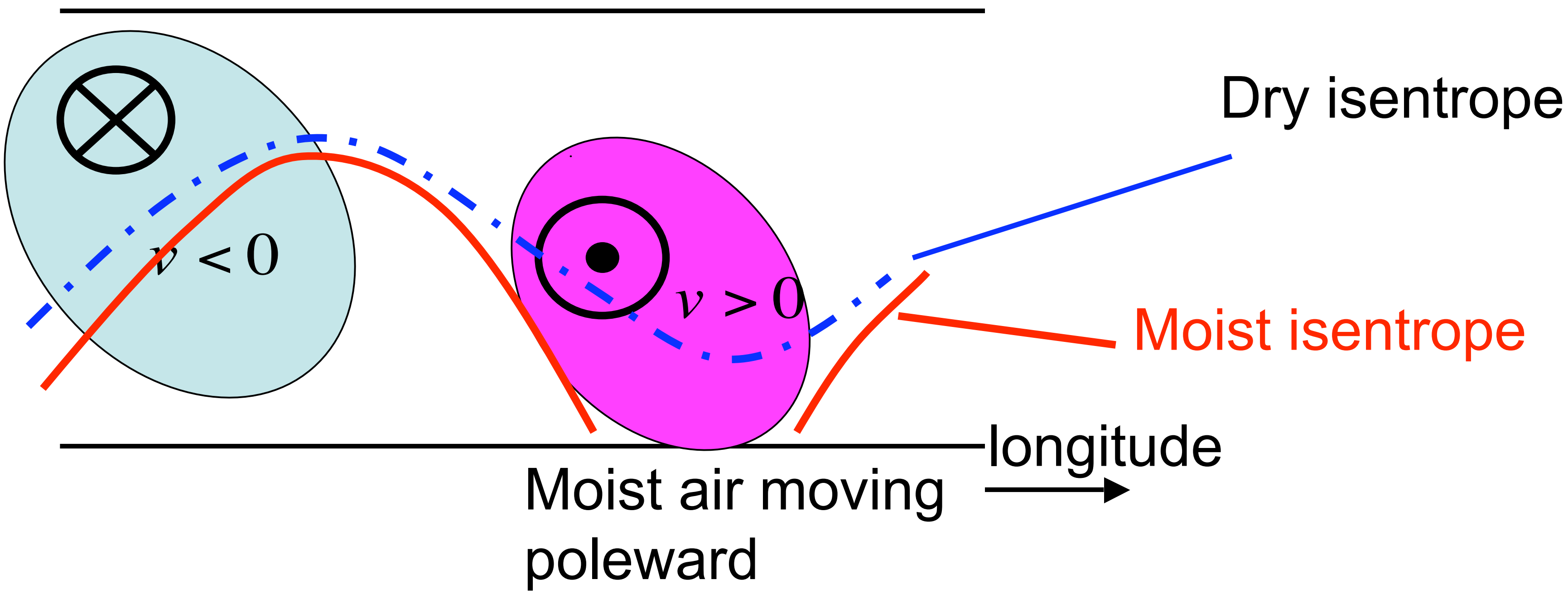


- The additional mass transport on moist isentropes takes place filaments near the Earth's surface.
- The equivalent potential temperature corresponds to upper tropospheric value of the potential temperature.
- This corresponds to a poleward flow of warm, moist air near the surface that is ready to rise into the upper troposphere.

Why the circulation on moist isentropes is larger?

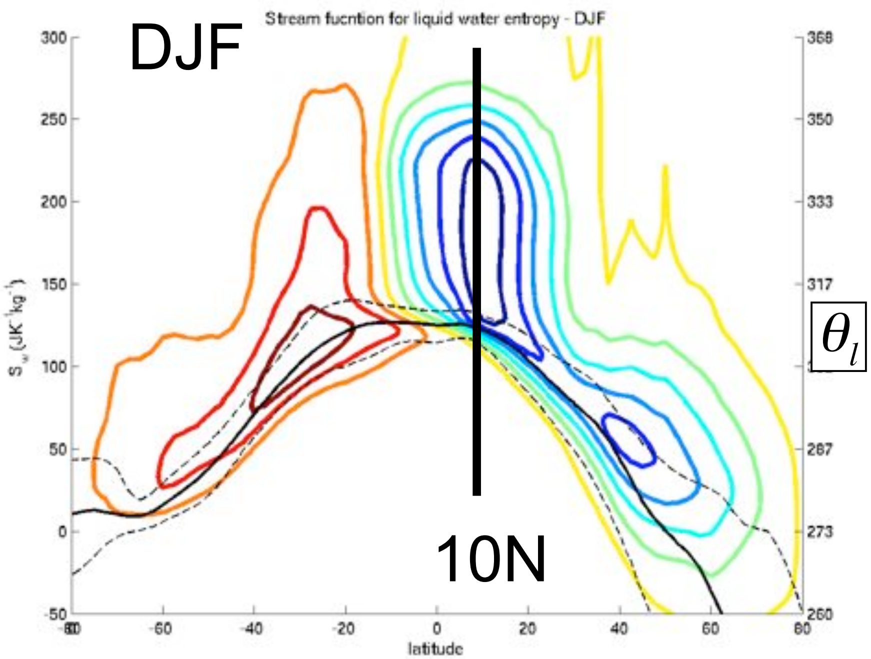


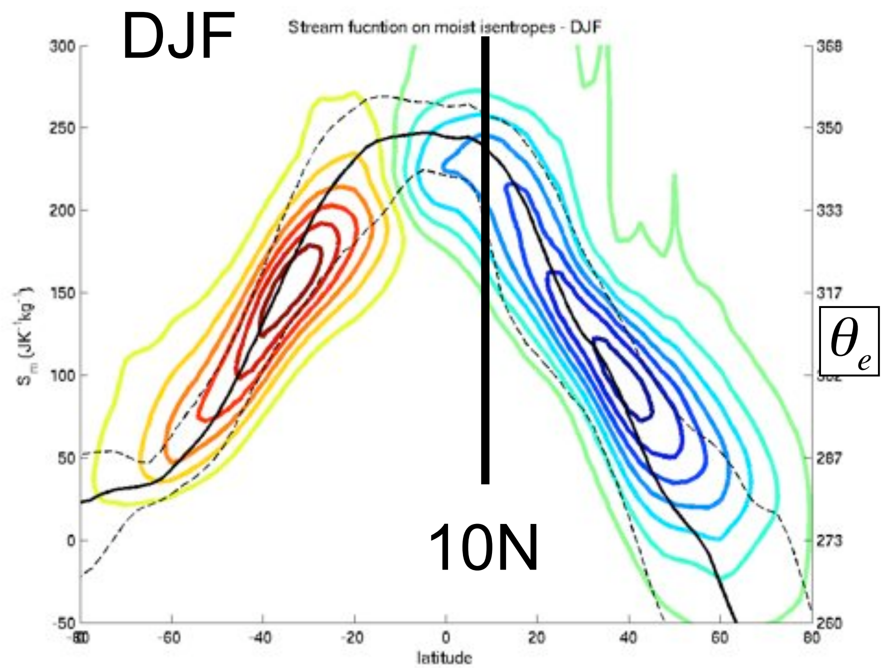
In the stormtracks:
Circulation on moist isentropes



- Moist isentropes found in the upper troposphere also intersects the Earth's surface.
- Such situation corresponds to a poleward flow of warm, moist air near the surface.

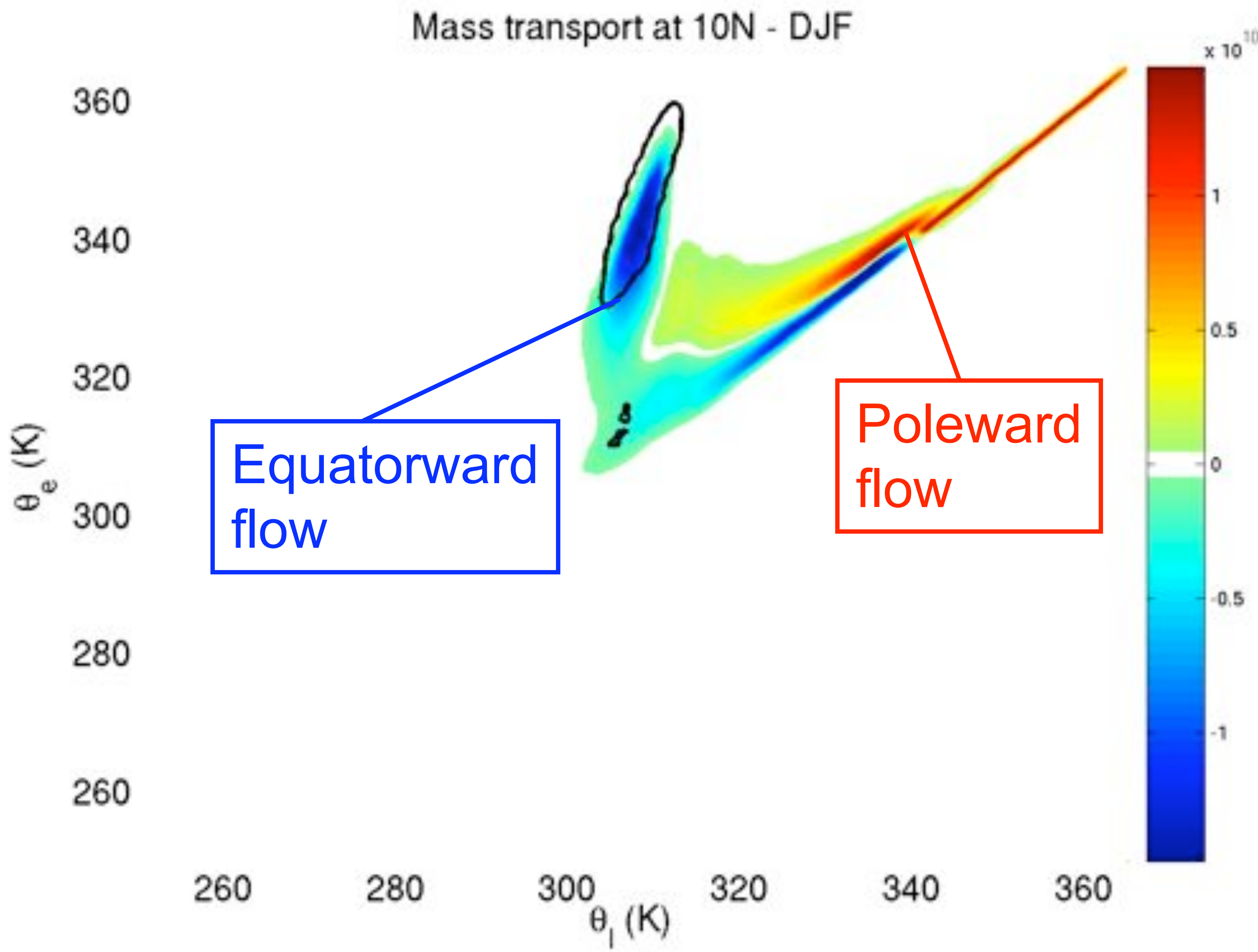
Hadley circulation:



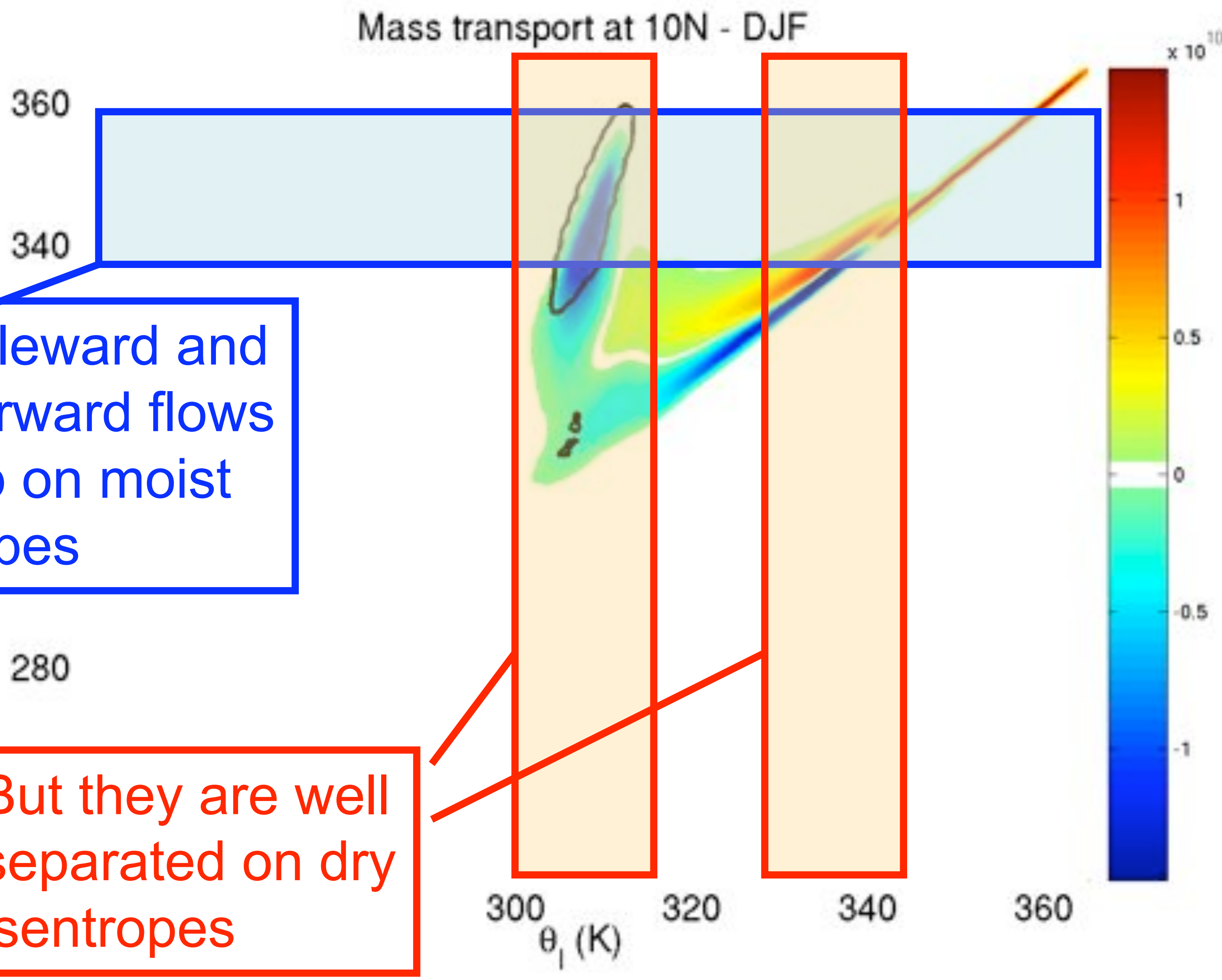


- Circulation on dry isentropes is larger than circulation on moist isentropes.

Mass flux distribution at 10N - DJF

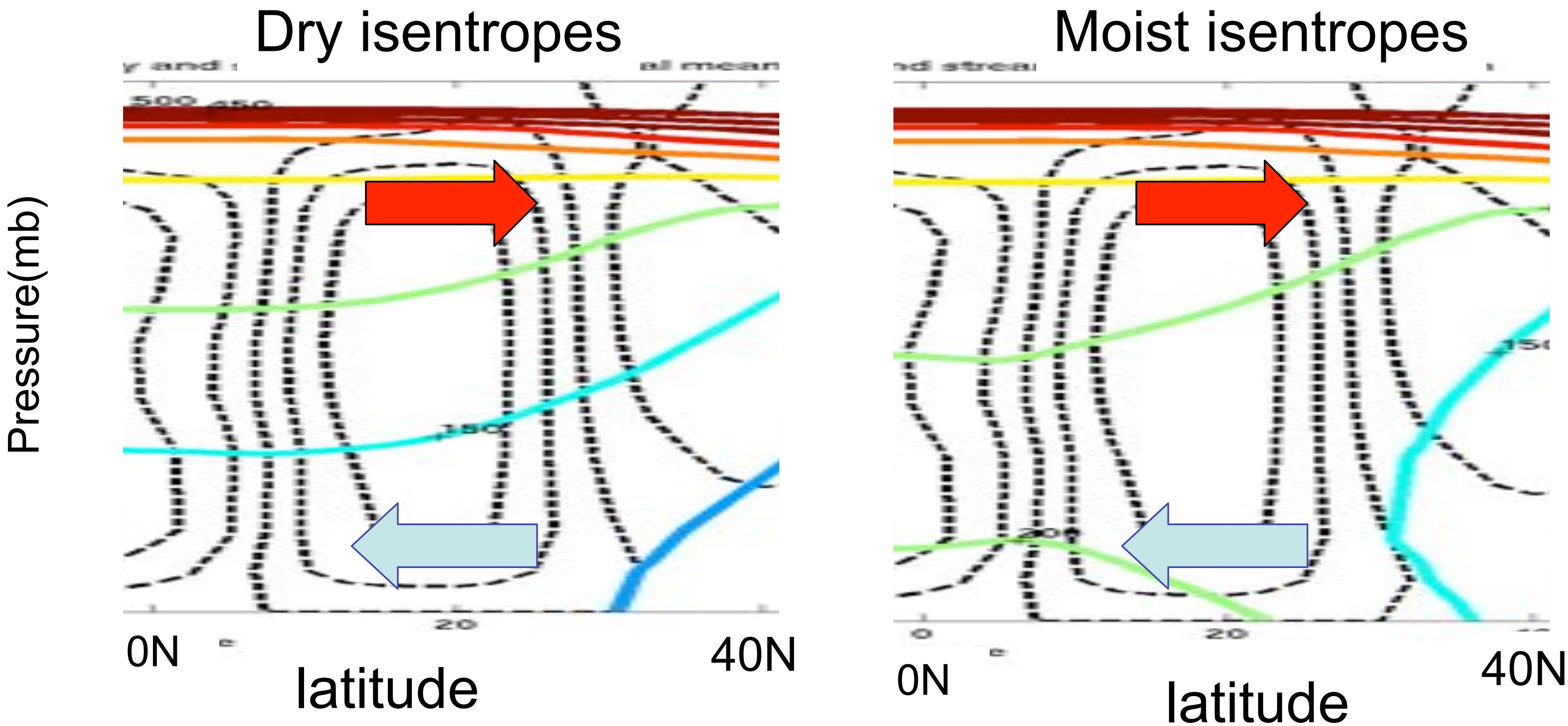


Mass flux distribution at 10N - DJF

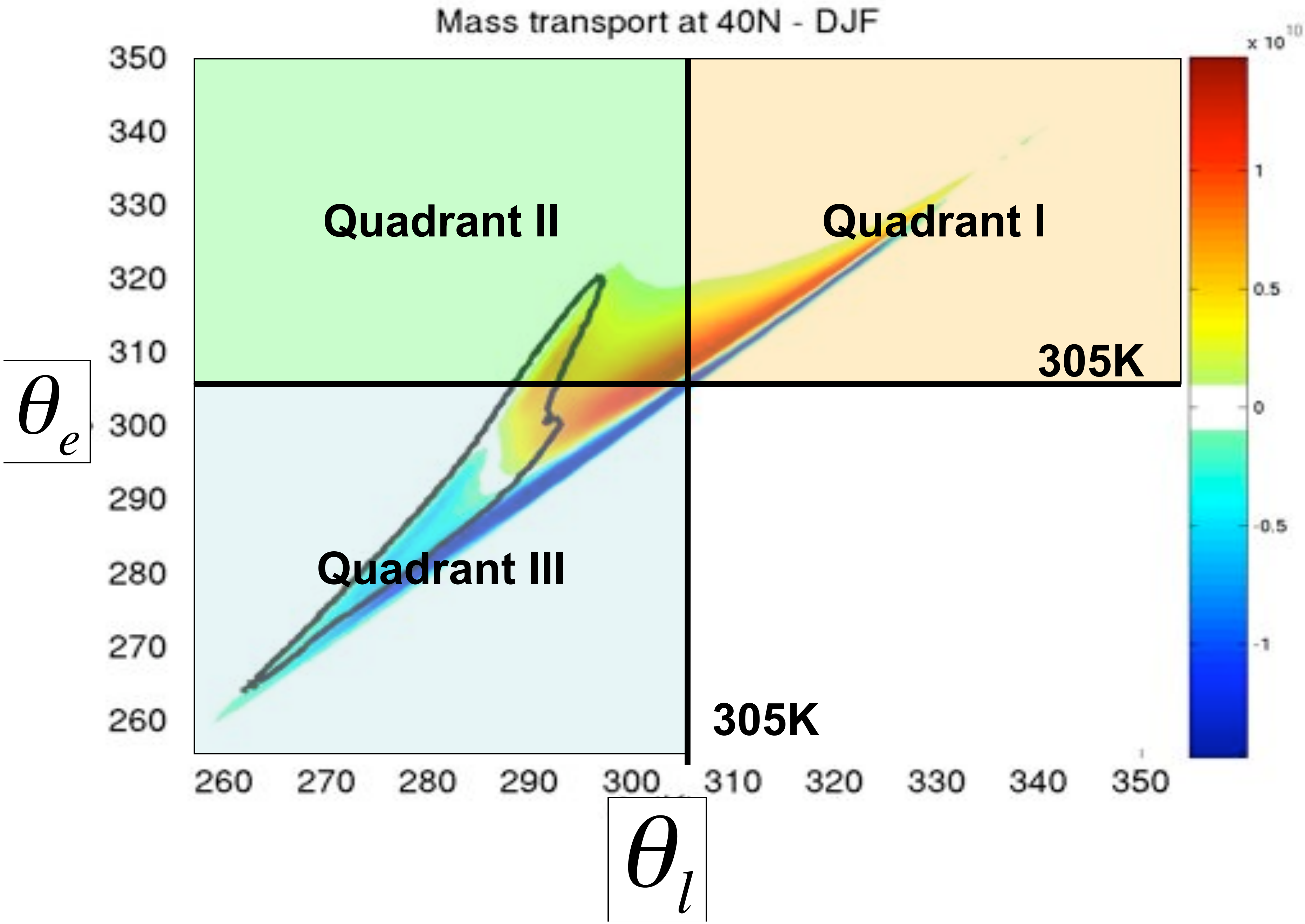


The poleward and equaotrward flows overlap on moist isentropes

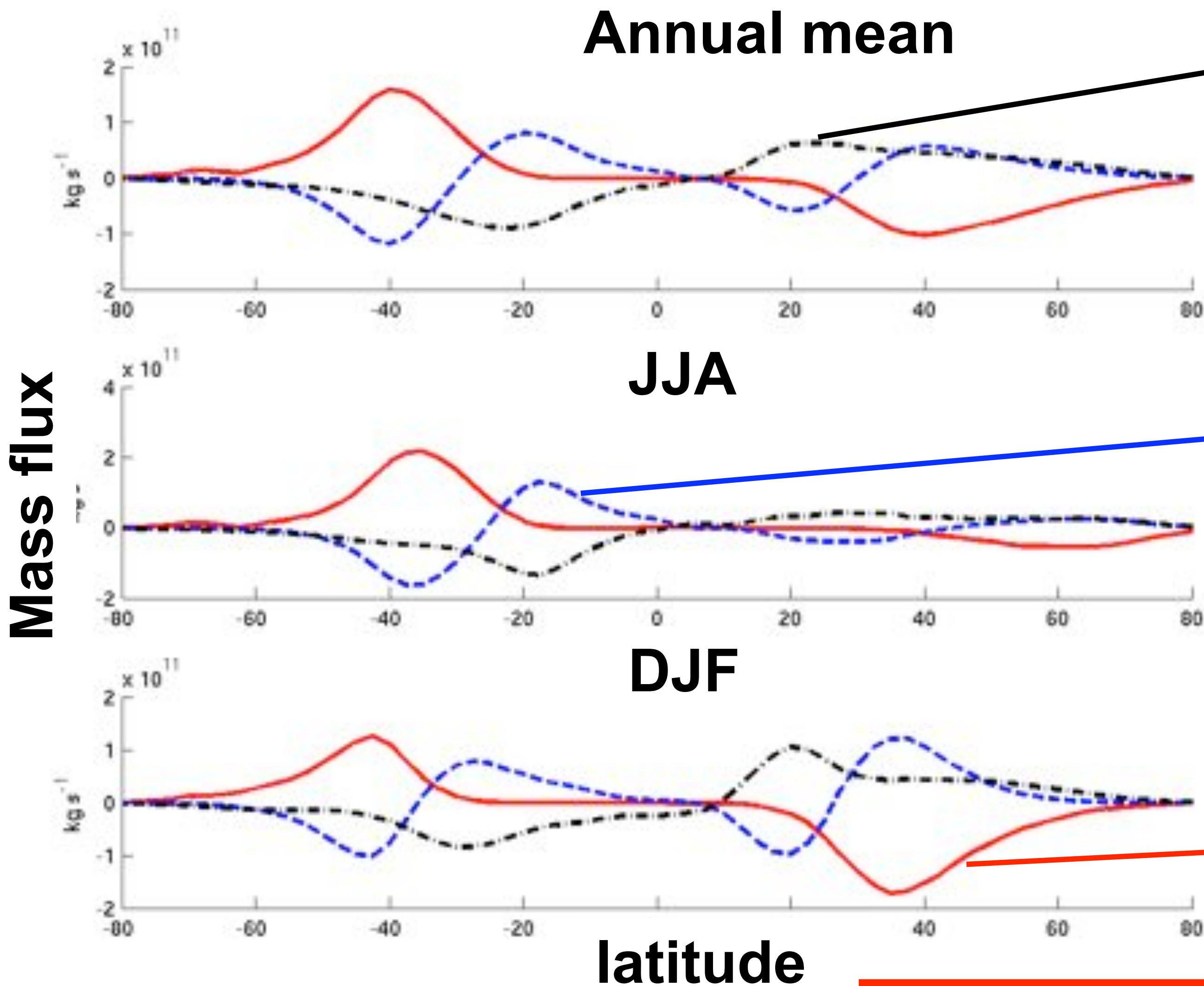
But they are well separated on dry isentropes



- In the equatorial regions, the potential temperature increases uniformly with height, but not the equivalent potential temperature.
- The equivalent potenital temperature in equatorward and poleward flows of the Hadley cell are close to each other.



Mass transport at 40N - DJF
Quadrant II
Quadrant I
Quadrant III
305K
305K
θ_e
θ_l
x 10^10
1
0.5
0
-0.5
-1
350
340
330
320
310
300
290
280
270
260
260
270
280
290
300
310
320
330
340
350



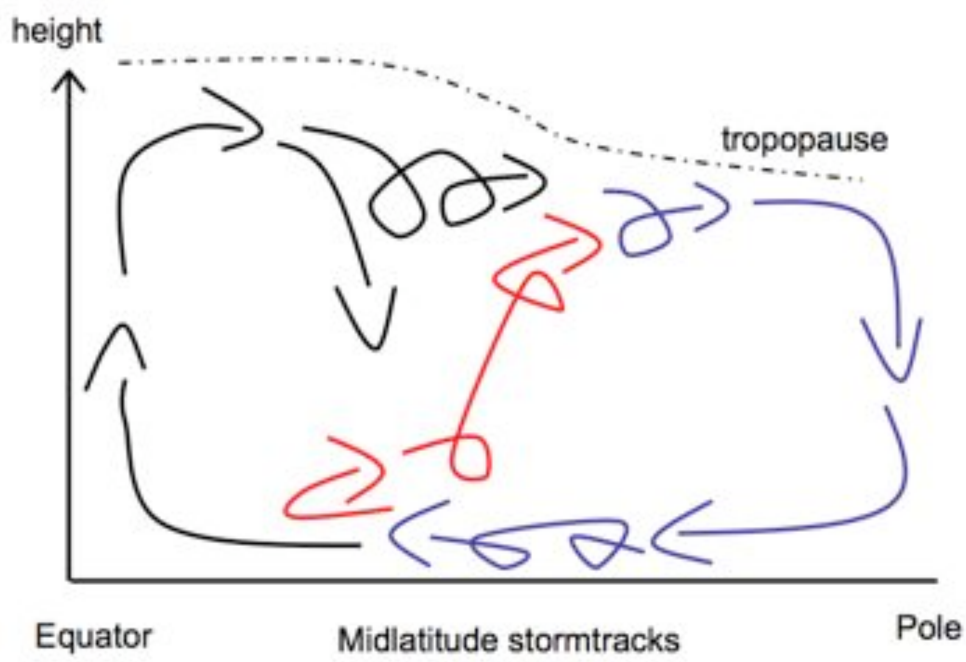
Quadrant I: Upper tropospheric flow

Quadrant II: Low level warm moist air

Quadrant III: Low level dry air

Tropics: upper tropospheric and moist air go in opposite direction

Midlatitudes: upper tropospheric and moist air go in the same direction

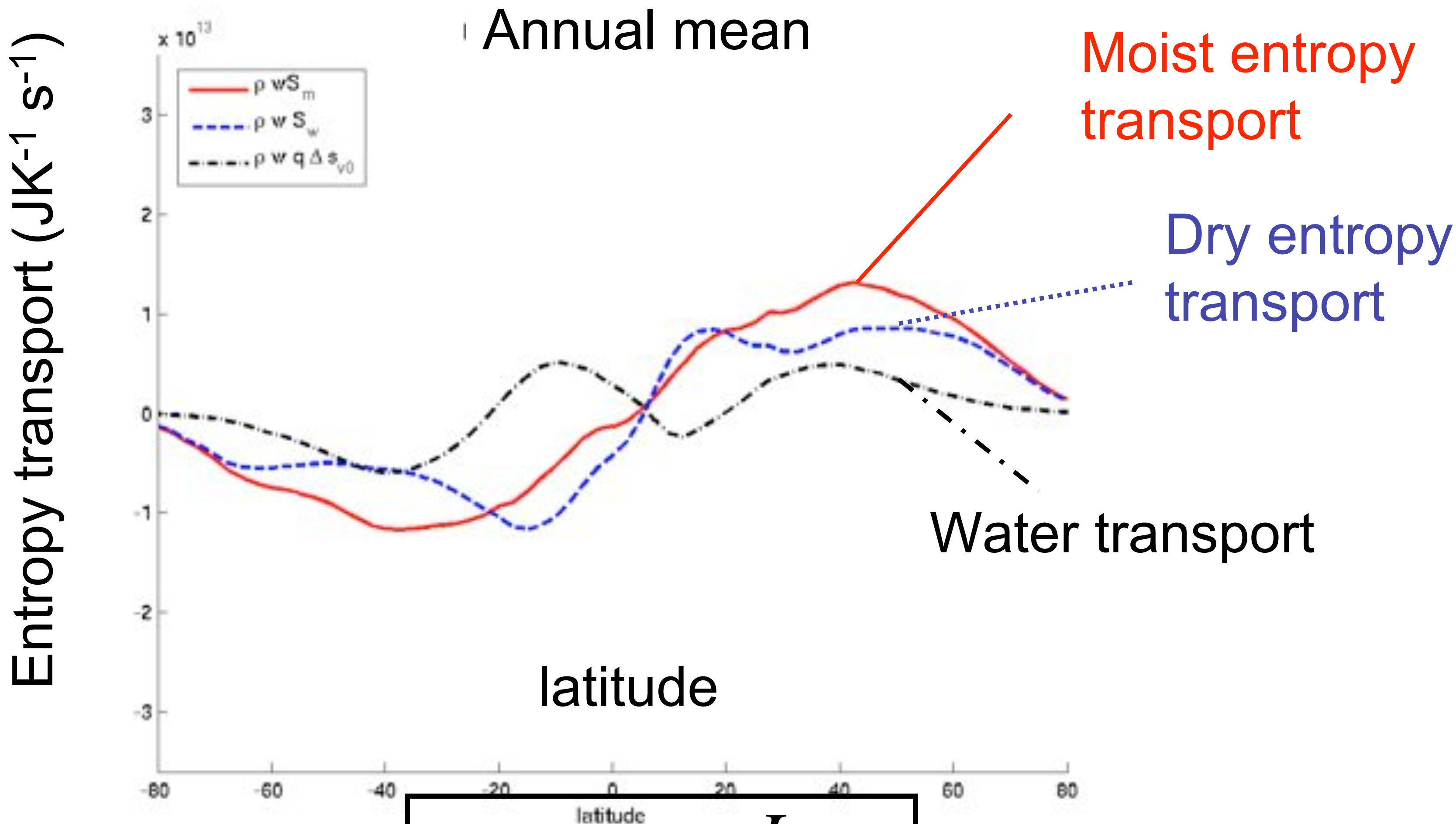


Circulation on dry isentropes

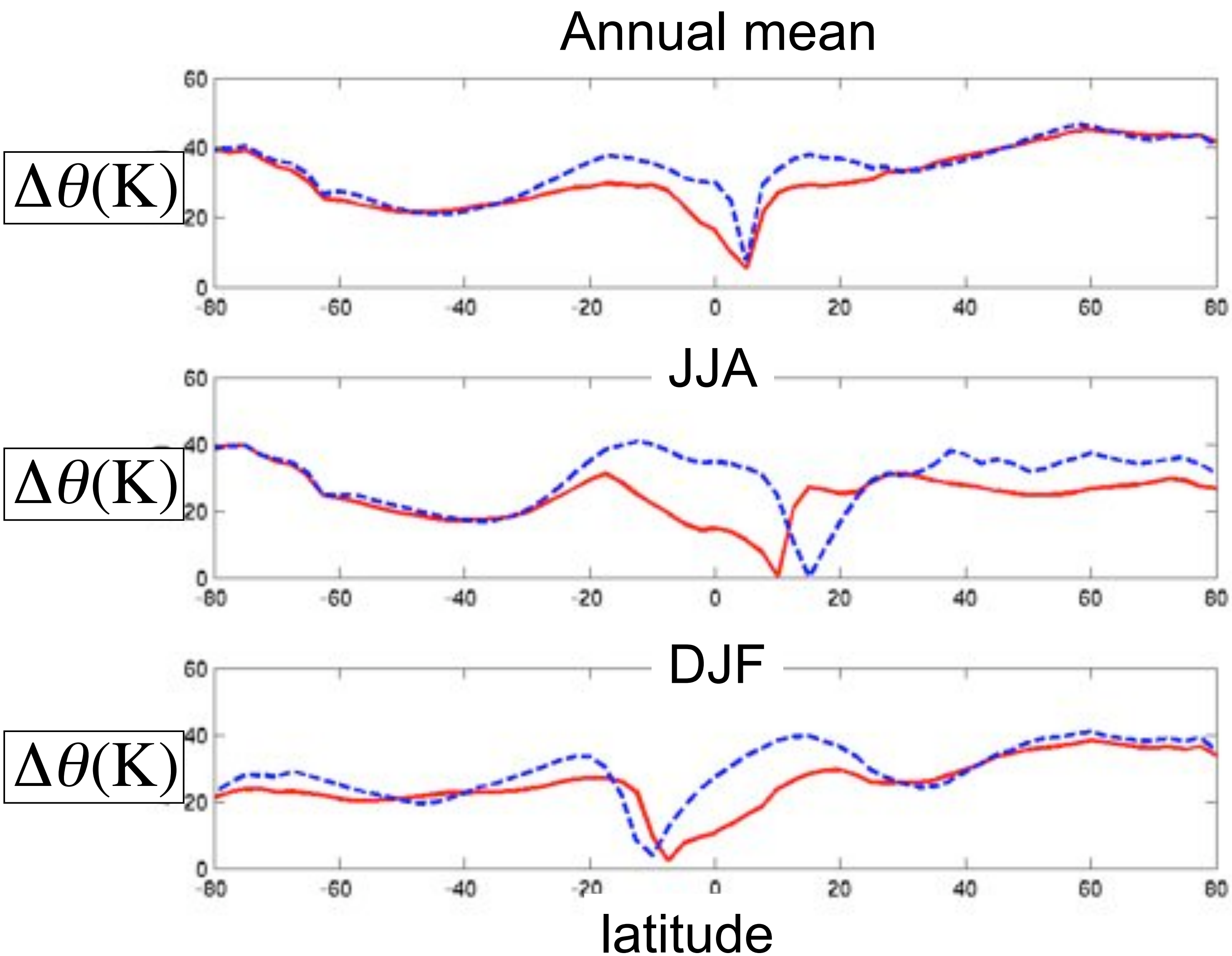
'Moist' branch: additional mass flow on moist isentropes

- The global circulation has two poleward components in the midlatitudes:
  - an upper tropospheric branch of high θ_e-θ_l;
  - an a lower branch of warm, most air with high θ_e-low θ_l, which ascent into the upper troposphere within the stormtracks.
- Mass transport is comparable in each branch.

$$\text{Entropy transport : } F_S = \int_0^{p_{surf}} \overline{vS}\,\frac{dp}{g}$$



$$F_{S_m} = F_{s_l} + \frac{L_v}{T_0} F_q$$



Gross Stratification:

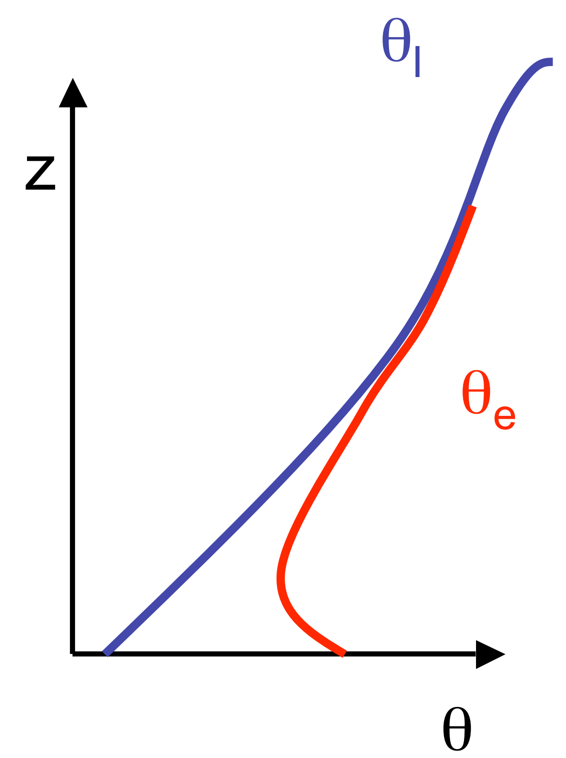
$$\Delta\theta = \frac{F_\theta}{\Delta\Psi_\theta}$$

~Entropy (or potential temperature) per unit mass transported

- In the tropics:

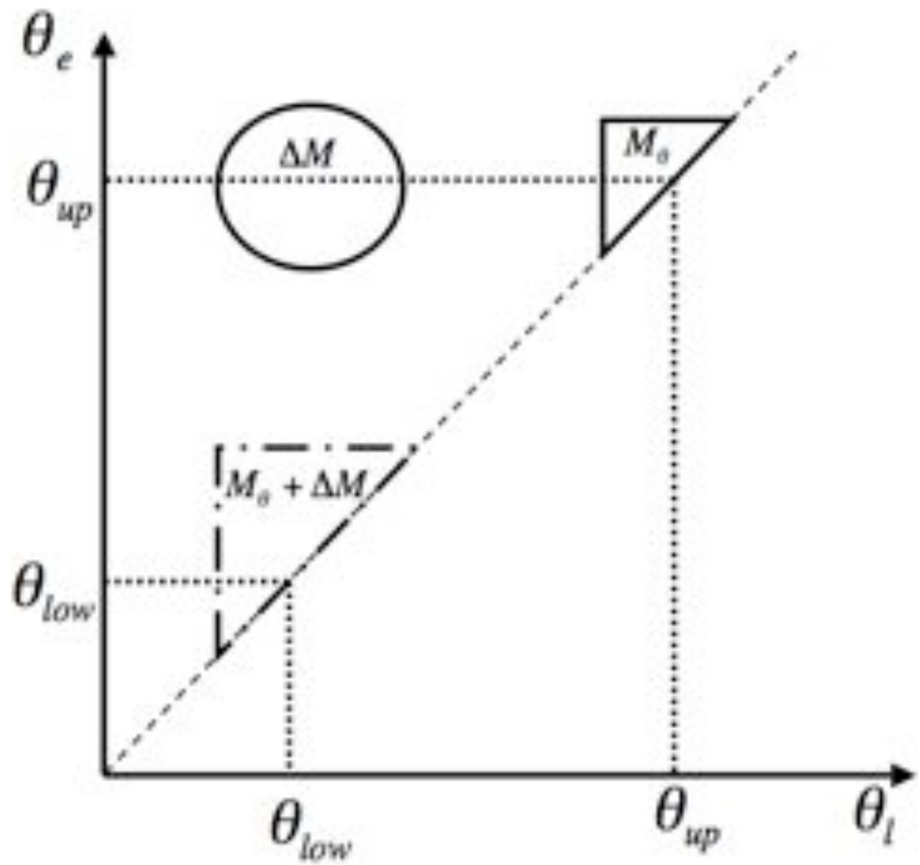
$$\Delta\theta_e << \Delta\theta_l$$

- Vertical stratification of humidity $\partial_z q$ is in opposite direction to that for potential temperature $\partial_z \theta$



In the midlatitudes: $\Delta\theta_e \sim \Delta\theta_l$

- In order to have enhanced mass transport but same stratification, the additional mass transport must take place at θ_l corresponding to lower tropospheric value, but θ_e corresponding to upper troposphere.
- It implies that horizontal variations of θ_e (and of water vapor) are comparable to its vertical variations.



Conclusions

- Heat engine and buoyancy flux computations yields the same answer.
- Entropy and buoyancy are closely tied:

$$\boxed{B' \propto \Gamma_{ad} S''}$$

- ‘Global mean circulation’ depends on the coordinate system - its is true also for any ‘isentropic’ circulation.
- In the midlatitudes, the mass transport on moist isentropes is approximately twice as large as that on dry isentropes.
- The additional mass transport corresponds to a low-level, poleward flow of warm, moist air that ascends into the upper troposphere within the stormtracks.

Further reading

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