Integration time in space tests of the Weak Equivalence Principle: why is it crucial?

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If you are really clever...

- At getting a driving signal as strong as possible...
- At implementing a read-out system adequate to reading the tiny target differential acceleration signal...
- At eliminating/attenuating/rejecting/measuring systematic errors...

...you will reach a point at which thermal noise must be faced and reduced below the target signal for the latter to emerge and be read.

Only very few experiments are so good to reach their thermal noise limit!

(Adelberger et al., PPNP 2009)

Signal strength

Strength of driving signal for WEP experiments on ground and in Low Earth Orbit (in $m\,s^{-2}$)

	Earth's field		*Sun's field*	
	Ground	*LEO*	*Ground*	*LEO*
mass dropping (*Galileo – like tests*)	9.8 → (*factor 1.2 loss*)	$\simeq 8$	—	—
suspended masses (*regardless of the suspension type : mechanic, electrostatic, superconducting coils...*)	$\simeq 0.016$ → (***factor 500 gain!***)	$\simeq 8$	$\simeq 0.0057$ (*factor 2.8 loss* from Earth's field Ground)	$\simeq 0.0057$

- Best mass dropping test: $7 \cdot 10^{-10}$ *(Carusotto et al. PRL, 1992)*
- Best suspended masses test
  - in the field of the Earth: $\simeq 10^{-13}$ *(Schlamminger et al. PRL, 2008)*
  - in the field of the Sun: 10^{-12} *(Baeßler et al. PRL, 1999)*

GG target in LEO: 10^{-17} (GG prototype is at: $8.9 \cdot 10^{-12}$ *Nobili et al., CQG 2012*)

Read-out

- It should be differential (provide good common mode rejection) because the sought for signal is differential
- It should have extremely low noise in the relevant bandwidth
- It should not affect negatively the thermal noise budget of the experiment. With capacitance read-out known issues to be taken care of are:
  - size of gap, because of residual gas damping (and patch effects)
  - loose conducting wire, because of internal damping

Systematics (I)

- We deal only with systematics we are aware of $\Downarrow$
  - study previous and other experiments very carefully
  - make search for competing effects as thorough as possible
  - get different people think about the experiment from different points of view
  - never discard criticisms…

  Hope that in so doing no crucial systematic error will go unnoticed…

- Eliminate/reduce systematics **by experiment design** whenever possible:
  e.g. radiometer effect (*Nobili et al. PRD rapid communication, 2001*) and mass anomalies of test bodies (*GG Phase A-2 Sudy, ASI 2009*) in GG

- Rely on their measurement (if they are not too large and/or measurement is accurate enough):
  e.g. direct measurement of patch effects in GG

Systematics (II)

- For the most dangerous systematics:
  - coupling of the monopole of the Earth with the quadrupole and multipole moments of the test masses
  - tidal effects

  identify carefully their signature, which is known and different from that of the signal (due to different dependence on orbital parameters and satellite/sensor attitude)

- If the integration time required to reach the target is short, many target-level measurements can be performed during a 9-month mission which allow these systematics to be discriminated with certainty from a possible violation signal. *This is celestial mechanics, not statistics… statistics is used only during the short integration time to reduce thermal noise…*

Nobili et al. in preparation

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Integration time: GG (I)

Nobili et al., to appear on PRD

According to the fluctuation-dissipation theorem any dissipative phenomenon has a fluctuating random force associated to it *(Callen & Welton PR, 1951; Nyquist PR, 1928)*:

$$< |\hat{F}_{th}(\omega)|^2 >= 4k_B T\gamma$$

If this force is larger than the signal it must be reduced by integrating (taking data) long enough:

$$t_{int} = SNR^2 \cdot \frac{< |\hat{F}_{th}(\omega_{signal})|^2 >_{tot}}{F(\omega_{signal})^2}$$

The integration time is inversely proportional to the signal force squared (in WEP tests it is the acceleration, not the force, that matters; the mass of test bodies helps reducing thermal noise and non gravitational effects in general. . . in GG 10 kg *each)*

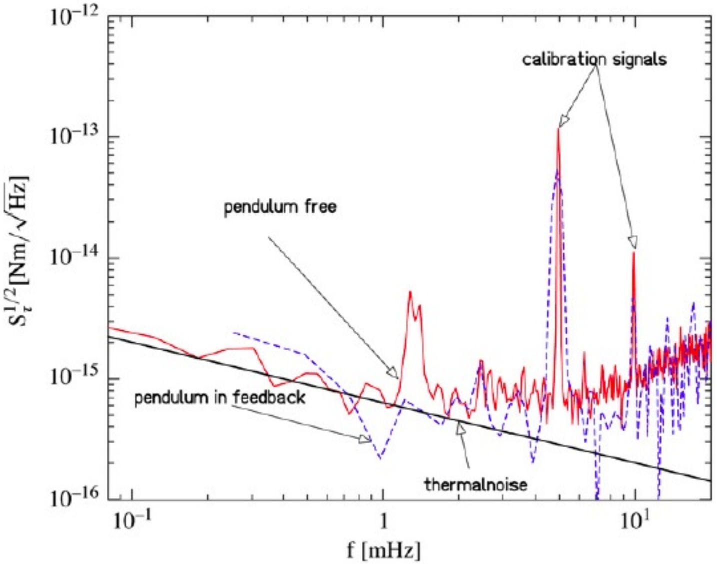
Integration time: GG (II)

$$< |\hat{F}_{th}(\omega_{signal})|^2 >_{tot} = \sum_i < |\hat{F}_{th}(\omega_{signal})_i|^2 >$$

- *i=1*: thermal noise due to internal (structural) damping γ_{id}
- *i=2*: thermal noise due to residual gas damping γ_{gas}
- *i=3*: thermal noise due to eddy currents damping γ_{eddy}

Integration time: GG (III)

E.G. Adelberger et al. / Progress in Particle and Nuclear Physics 62 (2009) 102–134



- Thermal noise due to internal damping usually dominant. Known to decrease with frequency (*Saulson PRD, 1990*):

$$\gamma_{id}(\omega) \simeq \frac{k\phi(\omega)}{\omega} = \frac{\mu\omega_n^2\phi(\omega)}{\omega}$$

  Better up-convert signal to higher frequency

- Demonstrated by Adelberger rotating the balance and up-converting the signal to the rotation frequency, ***just below the resonance frequency***. Above resonance, effects are attenuated like in any 1D oscillator, and read-out noise dominates

Integration time: GG (IV)

- No such attenuation occurs in 2D oscillators when the signal is up-converted by rotation above resonance

(*Pegna et al. PRL, 2011*; demonstrated experimentally in GG prototype *Nobili et al., CQG, 2012*).

- In GG rotation up-converts the signal from the orbital frequency to a region where thermal noise from internal damping is reduced by a very large factor:

$$\frac{<|\hat{F}_{th-id}(\omega_{orb})|^2>}{<|\hat{F}_{th-id}(\omega_{spin})|^2>} \gtrsim \frac{\omega_{spin}}{\omega_{orb}} \simeq 6000$$

down to (at $T \simeq 300\,K$ with $\Phi \simeq 1/20000$, $\omega_n \simeq 2\pi/540\,\mathrm{rad\,s^{-1}}$):

$$<|\hat{F}_{th-id}(\omega_{spin})|^2> \simeq 4k_B T \gamma_{id}(\omega_{spin}) \simeq 8.9 \cdot 10^{-29}\,\mathrm{N^2/Hz}$$

which turns out to be lower than thermal noise from residual gas damping

Integration time: GG (V)

$$< |\hat{F}_{th}(\omega_{spin})|^2 >_{tot} = < |\hat{F}_{th-gas}|^2 > + < |\hat{F}_{th-id}(\omega_{spin})|^2 > + < |\hat{F}_{th-eddy}|^2 > \simeq 2.2 \cdot 10^{-28} + 8.9 \cdot 10^{-29} + 6.5 \cdot 10^{-29}\,\mathrm{N^2/Hz} \simeq 3.74 \cdot 10^{-28}\,\mathrm{N^2/Hz}$$

- Gas damping noise estimated with reference to Cavalleri et al., PRL 2009 and a 2 cm gap as in GG baseline with laser gauge read-out.
- Eddy currents damping estimated assuming gradient of the Earth's magnetic field as large as the field itself and with a 150 reduction by μ-metal shield

With $SNR = 2$ and a WEP target to 10^{-17} (test bodies 10 kg each; $F_{signal} \simeq 4 \cdot 10^{-16}$ N) the required integration time is:

$$t_{int} = SNR^2 \cdot \frac{< |\hat{F}_{th}(\omega_{spin})|^2 >_{tot}}{F_{signal})^2} = 4 \cdot \frac{3.74 \cdot 10^{-28}}{(4 \cdot 10^{-16})^2} \simeq 2.7\,\mathrm{h}$$

A full 10^{-17} measurement will be done in 1 d (8 t_{int} cycles, almost 15 orbits)

Integration time: μSCOPE

According to μSCOPE scientists the dominant source of thermal noise is due to internal damping in the gold wire connecting each test mass to its enclosure. The SD of thermal acceleration noise is estimated to be
(Touboul Space Sci. Rev., 2009; Touboul et al. CQG, 2012):

$$a_{th-\mu scope} \simeq 1.4 \cdot 10^{-12}\,\mathrm{ms}^{-2}/\sqrt{\mathrm{Hz}}$$

For a WEP test to 10^{-15} and $SNR = 2$:

$$a_{WEP-\mu scope} \simeq 8 \cdot 10^{-15}\,\mathrm{ms}^{-2}$$

the resulting integration time is:

$$t_{int-\mu scope} = 4 \cdot \frac{(1.4 \cdot 10^{-12})^2}{(8 \cdot 10^{-15})^2} \simeq 1.4\,\mathrm{d}$$

which allows a reliable measurement in several days and leaves room for checks and/or improvements in 9-month mission.
Aiming to 100 times better would require a 10^4 times longer integration time!
Would cryogenics be the answer????

Integration time: Q-WEP

- Q-WEP: WEP test with dual species ^{85}Rb, ^{87}Rb to 10^{-14} on the ISS *(Study funded by ESA)*

Expected single shot differential acceleration sensitivity:

$$\frac{\sigma_{\Delta a}}{\sqrt{Hz}} = \frac{\sqrt{2}}{kT^2\sqrt{N}}\sqrt{T_C} \simeq 1.86 \cdot 10^{-10}\,\frac{\mathrm{ms}^{-2}}{\sqrt{Hz}}$$

$$(N = 10^6\ atoms, k = 8\pi/(780\,\mathrm{nm}) = 3.22 \cdot 10^7\,\mathrm{m}^{-1}, \mathrm{T_C} = 18\,\mathrm{s})$$

Crucial to achieving this sensitivity is a huge rejection of common mode vibration noise (to 10^8 - 10^9) to be achieved by very fine Rabi-frequency and k-vector matching. Tested on ground only for a single species gradiometer (McGuirk et al. PRA, 2002)

Several months of continuous "cycles" are needed to reach the target differential acceleration sensitivity $\Delta a_{QWEP} \simeq 8.7 \cdot 10^{-14}\,\mathrm{ms}^{-2}$ (i.e. to perform just 1 single measurement!!!)

Open issues:

- Is there any control on atom clouds Δh at release? (remember taht Δh mimics signal...)
- Since atoms of different species have different mass, is there any Δh bias? If so $\Rightarrow$ no longer a random process...
- Physical system over very long times???

A dual species AI can <u>and should</u> be tested on ground
Only way to establish where it stands and how much it could gain in space